Chapter 10

Simple Interest And Compound Interest

Sequence of the Chapter

- Simple Interest
- Compound Interest
- Computation Of Compound Interest When Interest Is Compounded Annually
- Computation Of Compound Interest When Interest Is Compounded Half-Yearly
- Computation Of Compound Interest When Interest Is Compounded Quarterly
- Computation Of Compound Interest By Using Formulae
- Growth And Depreciation

INTRODUCTION

In the previous classes, we have seen that when money is borrowed, extra money is paid to the lender for the use of the borrowed money for a given period of time at a given rate. The extra money paid is called **interest**. When money is lent or borrowed on **simple interest**, interest is calculated every year.

The money borrowed or lent is called the **principal**. The sum of the principal and the interest is called the **amount**. The rate at which interest is calculated is called the **rate** of interest per annum.

SIMPLE INTEREST

We know that while calculating simple interest, it is calculated uniformly throughout the loan period on the original principal and is given by the following formula :

$$\text{Simple Interest (S.I.)} = \frac{\text{Principal} \times \text{Rate} \times \text{Time}}{100}$$

$$\text{S.I.} = \frac{P \times R \times T}{100}$$

$$\text{Amount} = \text{Principal} + \text{S.I.}$$

Example 1 : Calculate the simple interest on ₹ 2,300 at 10% p.a. for 6 months.

Solution : $P =$ ₹ 2,300, $R = 10\%$ p.a.

$$T = 6 \text{ months} = \frac{6}{12} \text{ years} = \frac{1}{2} \text{ years}$$

$\therefore \quad \text{S.I.} = \dfrac{P \times R \times T}{100} = \dfrac{2{,}300 \times 10 \times 1}{100 \times 2} = ₹\,115.$

Example 2 : A person borrowed ₹ 5,000 for 1 year and ₹ 6,000 for $1\frac{1}{2}$ year at the same rate of interest. The total interest he had to pay is ₹ 2,800. Find the rate of interest.

Solution : **Case I.** **Let *x* be the rate of interest.**

$P = ₹\,5{,}000,\ T = 1\ \text{year},\ R = x$

$\therefore \quad \text{S.I.}_1 = \dfrac{P \times R \times T}{100} = \dfrac{5{,}000 \times x \times 1}{100} = 50x$

Case II. $\boldsymbol{P = ₹\,6{,}000,\ T = 1\frac{1}{2}\ \text{year} = \frac{3}{2}\ \text{year},\ R = x}$

$\therefore \quad \text{S.I.}_2 = \dfrac{P \times R \times T}{100} = \dfrac{6{,}000 \times x \times 3}{100 \times 2} = 90x$

Now, $\text{S.I.}_1 + \text{S.I.}_2 = 2{,}800$

$\therefore \quad 50x + 90x = 2{,}800$

$\Rightarrow \quad 140x = 2{,}800$

$\Rightarrow \quad x = \dfrac{2{,}800}{140} = 20\%$

Hence, rate of interest = 20%.

Example 3 : Kashish deposited a sum of ₹ 500 at the rate of 5% in the bank on August, 5. On October 17, she withdrew the sum deposited along with the interest. Find the amount she got.

Solution : $P = ₹\,500,\ T = \text{Aug. } 5 \text{ to Oct. } 17,\ R = 5\%$ p.a.

Now, Aug. 5 to Aug. 31 = 26 days

[Here, Aug. 5 is not included but Aug. 31 is included.]

Sept. = 30 days

Oct. = 17 days

Total days = 73 days = $\dfrac{73}{365}$ years

$\therefore \quad \text{S.I.} = \dfrac{P \times R \times T}{100} = \dfrac{500 \times 5 \times 73}{100 \times 365} = ₹\,5$

So, Kashish got amount = $P + I$ (S.I.) = ₹ 500 + ₹ 5 = ₹ 505.

COMPOUND INTEREST

In this method, the rate of interest and the time interval for compounding are fixed. The interest is calculated on the principal after the first time interval. Then it is compounded (added) to the principal. This amount then becomes the principal for the next time interval and so on. Compounding of interest allows a principal amount to grow at a faster rate than simple interest.

In the compound interest method, the rate of interest and time interval are fixed but the principal varies. In other words, if the borrower and the lender agree to a certain time interval (like a year or a half year or a quarter of a year or monthly or daily, etc.) and the rate of interest is also fixed, then Amount (Principal + Interest) at the end of the first time interval becomes the principal for the next time interval and so on. The total interest over the complete time period, so calculated is called the **compound interest (C.I.)**. To put it simply, the borrower is charged interest on previous interest also.

Conversion Period

While calculating compound interest, it is important to take note of the **period** or **conversion period**. This is the time period after which the interest is added to the principal (sum that was available at the beginning of the period) to get the new principal.

If this period is one year, then we say that the interest is **compounded annually** and the conversion period is **one**.

If the interest is added after every six months, we say that the interest is **compounded half-yearly** and the conversion period is **two** (since interest is added twice a year).

If the interest is added after every three months, we say that the interest is **compounded quarterly** and the conversion period is **four** (since interest is added four times in a year).

COMPUTATION OF COMPOUND INTEREST WHEN INTEREST IS COMPOUNDED ANNUALLY

Example 4 : Find the compound interest on ₹ 1,500 for 2 years at the rate of 5% per annum.

Solution : Principal for the 1st year = ₹ 1,500

Rate = 5%, Time = 1 year

Interest for the 1st year $= \frac{1{,}500 \times 5 \times 1}{100} =$ ₹ 75

Amount = ₹ 1,500 + ₹ 75 = ₹ 1,575

Principal for the 2nd year = ₹ 1,575

Time = 1 year, Rate = 5%

Interest for the 2nd year $= \frac{1575 \times 5 \times 1}{100} = \frac{315}{4} =$ ₹ 78.75

Compound interest for 2 years

= Interest for the 1st year + Interest for the 2nd year

= ₹ 75 + ₹ 78.75 = ₹ 153.75.

Example 5 : Find the amount on a sum of ₹ 5,000 for 2 years at the rate of 2% per annum and compound interest.

Solution : $P_1 =$ ₹ 5,000, $R = 2\%$ p.a., $T = 1$ year

$\therefore \quad I_1 = \frac{5{,}000 \times 2 \times 1}{100} =$ ₹ 100

$A_1 = P_1 + I_1 =$ ₹ 5,000 + ₹ 100 = ₹ 5,100

Now, $P_2 =$ ₹ 5,100, $R = 2\%$ p.a., $T = 1$ year

$$\therefore \quad I_2 = \frac{5{,}100 \times 2 \times 1}{100} = ₹\,102$$

$$A_2 = P_2 + I_2 = ₹\,5{,}100 + ₹\,102 = ₹\,5{,}202$$

Thus, amount at the end of the second year = ₹ 5,202 and compound interest

$= A - P =$ ₹ 5,202 – ₹ 5,000 = ₹ 202.

COMPUTATION OF COMPOUND INTEREST WHEN INTEREST IS COMPOUNDED HALF-YEARLY

If the rate of interest is r % per annum, then on compounding half-yearly, the rate of interest will become $\frac{r}{2}$% per half year. Also, the time is then converted in terms of half year. *For example,* if the time is n years, then we write it as $2 \times n$ half years.

Example 6 : Naveen takes a loan of ₹ 50,000 at 5% per annum for $1\frac{1}{2}$ years. Assuming interest to be compounded half-yearly, find the total interest paid by him.

Solution : $P =$ ₹ 50,000, $R = 5\%$, $T = 1\frac{1}{2}$ years (compounded half-yearly)

Interest at the end of six month $= 50{,}000 \times \frac{5}{100} \times \frac{1}{2} = 1{,}250$

Now, principal for the next six month = ₹ 50,000 + ₹ 1,250 = ₹ 51,250

Interest at the end of year $= 51{,}250 \times \frac{5}{100} \times \frac{1}{2} =$ ₹ 1,281.25

Now, principal for the next six month = ₹ 51,250 + ₹ 1,281.25 = ₹ 52,531.25

Interest at the end of $1\frac{1}{2}$ year $= 52{,}531.25 \times \frac{5}{100} \times \frac{1}{2} =$ ₹ 1,313.28(approx.)

Amount to be paid after $1\frac{1}{2}$ year = ₹ 52,531.25 + ₹ 1,313.28 = ₹ 53,844.53

Compound interest to be paid after $1\frac{1}{2}$ years = ₹ 53,844.53 – ₹ 50,000

= ₹ 3,844.53.

COMPUTATION OF COMPOUND INTEREST WHEN INTEREST IS COMPOUNDED QUARTERLY

If the rate of interest is r % per annum, then on compounding quarterly, rate of interest will become $\frac{r}{4}$% per quarter. Also, the time will be calculated in terms of quarters. *For example,* if the time is n years, then we write it as $4 \times n$ quarter years.

Example 7 : Find the compound interest on ₹ 6,000 for 1 year at the rate of 16% per annum compounded quarterly.

Solution : Rate of interest = 16% per annum $= \frac{16}{4}\% = 4\%$ per quarter, Time = 1 year = 4 quarters

Principal for the first quarter = ₹ 6,000, Rate of interest = 4%

Interest for the first quarter $= \frac{6,000 \times 4 \times 1}{100} =$ ₹ 240

$\therefore$ Amount at the end of the first quarter = ₹ 6,000 + ₹ 240 = ₹ 6,240

Principal for the second quarter = ₹ 6,240, Rate of interest = 4%

Interest for the second quarter $= \frac{6,240 \times 4 \times 1}{100} =$ ₹ 249.60

$\therefore$ Amount at the end of the second quarter = ₹ 6,240 + ₹ 249.60

= ₹ 6,489.60

Principal for the third quarter = ₹ 6,489.60, Rate of interest = 4%

Interest for the third quarter $= \frac{6,489.60 \times 4 \times 1}{100} =$ ₹ 259.584

$\therefore$ Amount at the end of the third quarter = ₹ 6,489.60 + ₹ 259.584

= ₹ 6,749.184

Principal for the fourth quarter = ₹ 6,749.184, Rate of interest = 4%

Interest for the fourth quarter $= \frac{6,749.184 \times 4 \times 1}{100} =$ ₹ 269.97

$\therefore$ Amount at the end of the fourth quarter = ₹ 6,749.18 + ₹ 269.97

= ₹ 7,019.15

$\therefore$ Compound Interest = ₹ 7,019.15 – ₹ 6,000 = ₹ 1,019.15.

Exercise : 10 (A)

1. Find the simple interest on ₹ 6,000 at 5% per annum for 3 years.

2. Mayank borrowed ₹ 8,450 from Nitin. At the end of 4 years Mayank had to pay back ₹ 12,506. What was the rate of interest?

3. Rajeev received a sum of ₹ 40,000 as a loan from a finance company at the rate of 7% per annum. Find the compound interest paid by Rajeev after 2 years.

4. A man deposits ₹ 1,000 in a savings bank account. How much will it amount in three years if the rate of interest is 5% per annum and the interest is payable annually?

5. Find the compound interest on ₹ 10,000 for $1\frac{1}{2}$ years at 10% per annum, interest being payable half-yearly.

6. Find the compound interest on ₹ 5,000 at the rate of 8% per annum for 1 year when the interest is compounded quarterly.

COMPUTATION OF COMPOUND INTEREST BY USING FORMULAE

The method we have used for the calculation of compound interest in the problems discussed so far is quite lengthy and cumbersome. As the number of periods for which interest is payable increases, the process of calculating the compound interest becomes more lengthy and time consuming. Therefore, we should look for a shorter method or a formula for calculating the compound interest.

Formula 1 : Let P be the principal and the rate of interest be $R\%$ per annum. If the interest is compounded annually, then the amount A and the compound interest C.I. at the end of n years are given by

$$A = P\left(1+\frac{R}{100}\right)^n$$

$$\text{C.I.} = A - P = P\left\{\left(1+\frac{R}{100}\right)^n - 1\right\}$$

Example 8 : Find the compound interest on ₹ 11,000 for 2 years at 10% per annum compounded annually.

Solution : $P = ₹\ 11{,}000,\ R = 10\%$ p.a.

$n = 2$ years

$$A = P\left(1+\frac{R}{100}\right)^n$$

$$= 11{,}000 \times \left(1+\frac{10}{100}\right)^2$$

$$= 11{,}000 \times \left(\frac{11}{10}\right)^2$$

$$= 11{,}000 \times \frac{11}{10} \times \frac{11}{10} = ₹\ 13{,}310$$

Compound Interest = Amount − Principal

= ₹ 13,310 − ₹ 11,000 = ₹ 2,310

Hence, compound interest is ₹ 2,310.

Example 9 : In how many years will a sum of ₹ 1,200 amount to ₹ 1,323 at the compound interest rate of 5% per annum?

Solution : $P = ₹\ 1{,}200,\ A = ₹\ 1{,}323,\ R = 5\%,\ n = ?$

$$A = P\left(1+\frac{R}{100}\right)^n$$

$$1{,}323 = 1{,}200\left(1+\frac{5}{100}\right)^n$$

$$\frac{1{,}323}{1{,}200} = \left(\frac{21}{20}\right)^n$$

$$\Rightarrow \frac{441}{400} = \left(\frac{21}{20}\right)^n$$

$$\Rightarrow \left(\frac{21}{20}\right)^2 = \left(\frac{21}{20}\right)^n$$

$\therefore \quad n = 2$

Hence, required number of years is 2 years.

Formula 2 : Let P be the principal and the rate of interest be R% per annum. If the interest is compounded half-yearly, quarterly, etc., then the amount A and the compound interest C.I. at the end of n years are given by

$$A = P\left(1 + \frac{R}{100k}\right)^{nk}$$

$$\text{C.I.} = A - P = P\left\{\left(1 + \frac{R}{100k}\right)^{nk} - 1\right\}$$

Here, interest is payable k times in a year.

Particular Cases

Case 1. When interest is compounded half-yearly or semi-annually

In this case, $k = 2$.

$$\therefore \quad A = P\left(1 + \frac{R}{2 \times 100}\right)^{2n}$$

$$\text{C.I.} = P\left\{\left(1 + \frac{R}{2 \times 100}\right)^{2n} - 1\right\}$$

Case 2. When interest is compounded quarterly

In this case, $k = 4$

$$\therefore \quad A = P\left(1 + \frac{R}{4 \times 100}\right)^{4n}$$

$$\text{C.I.} = P\left\{\left(1 + \frac{R}{4 \times 100}\right)^{4n} - 1\right\}$$

Example 10 : Kunal took a loan of ₹ 3,90,625 from a private financer. If the financer charges interest at the rate of 16% per annum, compounded quarterly. What amount will discharge Kunal's debt after 1 year? Also find the compound interest on the principal.

Solution : $P =$ ₹ 3,90,625, $R = 16\%$ per annum, $n = 1$ year

$$\therefore \quad \text{Amount } (A) \text{ after 1 year} = P\left(1 + \frac{R}{4 \times 100}\right)^{4n}$$

$$= 3{,}90{,}625\left(1 + \frac{16}{400}\right)^{4 \times 1} = 3{,}90{,}625\left(1 + \frac{1}{25}\right)^{4}$$

$$= 3{,}90{,}625 \times \left(\frac{26}{25}\right)^{4}$$

$$= 3{,}90{,}625 \times \frac{26}{25} \times \frac{26}{25} \times \frac{26}{25} \times \frac{26}{25} = ₹\ 4{,}56{,}976$$

An amount of ₹ 4,56,976 will discharge Kunal from his debt after 1 year.

∴ C.I. $= A - P =$ ₹ 4,56,976 − ₹ 3,90,625 = ₹ 66,351.

Formula 3 : Let P be the principal and $R_1\%$, $R_2\%$, $R_3\%$,..........., $R_n\%$ be the rates of interest for 1st, 2nd, 3rd, ..., nth year respectively. Then,

$$A = P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)\left(1+\frac{R_3}{100}\right)\cdots\cdots\cdots\left(1+\frac{R_n}{100}\right)$$

$$\text{C.I.} = P\left\{\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)\left(1+\frac{R_3}{100}\right)\cdots\cdots\left(1+\frac{R_n}{100}\right)-1\right\}$$

Example 11 : Find the amount and compound interest on ₹ 12,500 for two years if the interest is compounded annually and the rate of interest is 15% for the first year and 16% for the second year.

Solution : $P =$ ₹ 12,500, $R_1 = 15\%$ p.a., $R_2 = 16\%$ p.a.

$$A = P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)$$

$$= 12{,}500\times\left(1+\frac{15}{100}\right)\times\left(1+\frac{16}{100}\right)$$

$$= 12{,}500\times\left(\frac{23}{20}\right)\times\left(\frac{29}{25}\right)$$

$$= 625\times 23\times\left(\frac{29}{25}\right)$$

$= 25\times 23\times 29 =$ ₹ 16,675

C.I. $= A - P$

= ₹ 16,675 − ₹ 12,500 = ₹ 4,175.

Formula 4 : Let P be the principal and R be the rate of interest per annum. When the interest is compounded annually but the time is in fraction, *i.e.*, time is $n\frac{l}{m}$ years, then

$$A = P\left(1+\frac{R}{100}\right)^n\left(1+\frac{\frac{lR}{m}}{100}\right)$$

$$\text{C.I.} = P\left\{\left(1+\frac{R}{100}\right)^n\left(1+\frac{\frac{lR}{m}}{100}\right)-1\right\}$$

Example 12 : Find the amount and compound interest on ₹ 50,000 at 10% per annum for $2\frac{1}{4}$ years.

Solution : $P = ₹\,50{,}000, R = 10\%, \text{Time} = 2\frac{1}{4}$ years

$$\text{Amount } (A) \text{ after } 2\frac{1}{4} \text{ years} = P\left(1+\frac{R}{100}\right)^2\left(1+\frac{\frac{1}{4}R}{100}\right)$$

$$= 50{,}000 \times \left(1+\frac{10}{100}\right)^2 \times \left(1+\frac{\frac{1}{4}\times 10}{100}\right)$$

$$= 50{,}000 \times \left(1+\frac{1}{10}\right)^2\left(1+\frac{1}{40}\right)$$

$$= 50{,}000 \times \left(\frac{11}{10}\right)^2 \times \left(\frac{41}{40}\right)$$

$$= 50{,}000 \times \frac{11}{10} \times \frac{11}{10} \times \frac{41}{40} = ₹\,62{,}012.5$$

$$\text{C.I.} = A - P = ₹\,62{,}012.5 - ₹\,50{,}000 = ₹\,12{,}012.5.$$

Exercise : 10 (B)

1. Find the amount and the compound interest. On ₹ 12,000 for 3 years at 6% per annum.

2. Find the compound interest at 5% per annum for 3 years on the principal, which gives simple interest of ₹ 2,400 at the same rate and for the same time.

3. The simple interest on a certain sum of money at the rate of 4% per annum for 3 years is ₹ 1,200. Find the compound interest on the same amount at the rate of 10% per annum for 3 years.

4. A sum of ₹ 25,000 invested at 8% compounded semi-annually amounts to ₹ 28,121.60. Compute the time period.

5. Find the compound interest on ₹ 2,000 for $1\frac{1}{2}$ years at 10% p.a., when the interest is compounded half-yearly.

6. Find the amount and compound interest on ₹ 16,000 at 15% per annum for $2\frac{1}{3}$ years.

7. Find the amount and compound interest on ₹ 6,000 in 3 years if the rate of interest is 5% for the first year, 4% for the second year and 3% for the third year.

8. At what rate per cent per annum will a sum of ₹ 4,000 yield a compound interest of ₹ 410 in 2 years?

GROWTH AND DEPRECIATION

The concept of compound interest is also applicable to the problems on the rate of growth. The rate of growth can be positive as well as negative. *For example,* it is positive in case of population growth and it is negative in case of depreciation in the value of machinery or decrease in the value of property, etc., over a period of time.

★ The relative increase in value is called 'appreciation' and the appreciation per unit of time is called the 'rate of appreciation'.

★ The relative decrease in asset value over a period of time is called 'depreciation' and the depreciation per unit of time is called the 'rate of depreciation'.

When the rate of growth is positive, R is taken as positive and if the rate of growth is negative, R is taken as negative and the formula changes accordingly, *i.e.*,

$A = P\left(1 + \frac{R}{100}\right)^n$, when R is positive, *i.e.*, growth rate is positive.

$A = P\left(1 - \frac{R}{100}\right)^n$, when R is negative, *i.e.*, growth rate is negative.

Example 13 : The population of a town is 50,000. If the annual birth rate is 5% and the annual death rate is 3%, find the population after two years.

Solution : Annual birth rate = 5%, Annual death rate = 3%

∴ Annual growth = (5 – 3)% = 2%
Initial population (P) = 50,000,
Rate of growth (R) = 2% p.a.,
Time (n) = 2 years

∴ Population after 2 years $= P\left(1 + \frac{R}{100}\right)^n$

$$= 50{,}000 \times \left(1 + \frac{2}{100}\right)^2$$

$$= 50{,}000 \times \left(1 + \frac{1}{50}\right)^2$$

$$= 50{,}000 \times \left(\frac{51}{50}\right)^2$$

$$= 50{,}000 \times \frac{51}{50} \times \frac{51}{50} = 52{,}020$$

Hence, the population of the town after 2 years will be 52,020.

Example 14 : A scooter was bought at ₹ 42,000. Its value depreciated at the rate of 8% per annum. Find its value after 1 year.

Solution : Present value of scooter,
$P =$ ₹ 42,000, $R = 8\%$ p.a., $n = 1$ year

∴ Value of scooter after 1 year $= P\left(1 - \frac{R}{100}\right)^n$

$$= 42{,}000 \times \left(1 - \frac{8}{100}\right)^1$$

$$= 42{,}000 \times \left(\frac{23}{25}\right) = 1{,}680 \times 23 = ₹\ 38{,}640$$

Hence, the value of the scooter after one year is ₹ 38,640.

Example 15 : A new computer costs ₹ 80,000. Due to advancement in technology, its value depreciates every year. If the rates of depreciation are 4%, 5% and 10% in three successive years, find its value after 3 years.

Solution : Original value of the computer $(P) = ₹\ 80{,}000$

Rate of depreciation in the first year $(R_1) = 4\%$

Rate of depreciation in the second year $(R_2) = 5\%$

Rate of depreciation in the third year $(R_3) = 10\%$

Let the depreciated value be A.

$$\therefore \quad A = 80{,}000\left(1-\frac{4}{100}\right)\left(1-\frac{5}{100}\right)\left(1-\frac{10}{100}\right)$$

$$= 80{,}000\left(1-\frac{1}{25}\right)\left(1-\frac{1}{20}\right)\left(1-\frac{1}{10}\right)$$

$$= 80{,}000 \times \frac{24}{25} \times \frac{19}{20} \times \frac{9}{10} = ₹\ 65{,}664$$

Hence, the depreciated value of the computer after 3 years is ₹ 65,664.

Exercise : 10 (C)

1. The population of a village is 64,000. What will be the population after 2 years, if it is increasing at the rate of 5% annually?

2. The population of a city is 2,50,000. If the annual birth rate is 6.5% and the annual death rate is 4.5%, what will be the population of the city after 3 years?

3. Present value of a LCD T.V. is ₹ 64,000. If it depreciates at the rate of 2.5 paise per rupee per annum, find the value of the T.V. after 3 years. By how much has the value of the T.V. gone down?

4. In a factory, the production of televisions rose from 6,400 units to 8,100 units in 2 years. Find the rate of growth per annum.

5. To complete a project in years, 7,500 workers were employed. At the end of the first year 15% workers were retrenched. At the end of the second year 20% of those working at that time were retrenched. In order to complete the project on time, the number of workers was increased by 15% at the end of the third year. How many workers were working during the fourth year?

6. Puneet started a business with an investment of ₹ 1,50,000. The first year he incurred a loss of 5% per annum and the second year a profit at 10% per annum. Find his net worth at the end of two years.

SUMMARY OF THE CHAPTER

- The money borrowed or lent is called the 'principal'.
- The extra money paid is called 'interest'.
- The sum of the principal and the interest is called the 'amount'.
- The rate at which interest is calculated is called the 'rate' of interest per annum.
- In the compound interest method, the rate of interest and the time interval are fixed but the principal varies.
- The relative increase in value is called 'appreciation' and the relative decrease in asset value over a period of time is called 'depreciation'.

Review Of The Chapter
(Task For Summative Assessment)

1. Sania deposited ₹ 18,000 in a bank for 3 years which pays 6% interest per year. Find the simple interest she will get at the end of 3 years.

2. Find the compound interest at the rate of 5% per annum for 3 years on that principal which is 3 years yields the simple interest of ₹ 1,200 at the rate of 5% per annum.

3. Find the compound interest in each of the following using the formulae :

(a) $P =$ ₹ 5,000, $R = 5$ paise per rupee per annum, $n = 3$ years

(b) $P =$ ₹ 20,000, $R = 20\%$ per annum compounded quarterly, $n = 1$ year

4. Compute the compound interest on ₹ 12,000 for $1\frac{1}{2}$ years at 10% per annum.

5. Find the difference in compound interest on ₹ 6,000 for 1 year at the rate of 4% per annum if in the first case interest is paid half-yearly and in the second case quarterly.

6. In a laboratory, the count of bacteria in a certain experiment was increasing at the rate of 2.5% per hour. Find the bacteria at the end of two hours if the count initially was 5,06,000.

Multiple Choice Questions (MCQs)
(Task For Formative Assessment)

1. At what rate of compound interest will ₹ 25,000 become ₹ 36,000 in two years?
(a) 10% p.a. ☐ (b) 20% p.a. ☐ (c) 30% p.a. ☐ (d) 40% p.a. ☐

2. The S.I. and C.I. for 1 year on an item is :
(a) equal ☐ (b) C.I. > S.I. ☐ (c) S.I. > C.I. ☐ (d) none of these ☐

3. The number of quarters in $3\frac{1}{2}$ years is :
(a) 4 ☐ (b) 8 ☐ (c) 12 ☐ (d) 14 ☐

4. The principal if compound interest is ₹ 331, rate is 10% p.a. compounded annually and time is 3 years, is :
(a) 500 ☐ (b) 1,000 ☐ (c) 5,000 ☐ (d) none of these ☐

5. If the interest is compounded half-yearly, then to find the amount, we ________ the given time.
(a) half ☐ (b) double ☐ (c) triple ☐ (d)make no change ☐

6. The difference between compound interest compounded annually and simple interest on a certain sum of money for 2 years at 5% p.a. is ₹ 12.50. What is compound interest on this sum for 2 years?
(a) ₹ 250 ☐ (b) ₹ 262.50 ☐ (c) ₹ 512.50 ☐ (d) ₹ 525.00 ☐

7. The compound interest on ₹ 1,000 for 2 years at the rate of 10% per annum compounded annually is :
(a) ₹ 180 ☐ (b) ₹ 210 ☐ (c) ₹ 550 ☐ (d) none of these ☐

8. $\left[P\left\{\left(1+\frac{R}{100}\right)^n - 1\right\}\right]$ is equal to :
(a) S.I. ☐ (b) Principal ☐ (c) Amount ☐ (d) C.I. ☐

9. The population of a place increased to 54,000 in 2007 at the rate of 5% per annum. The population in 2005 was :
(a) 45,370 ☐ (b) 47,360 ☐ (c) 48,735 ☐ (d) 48,980 ☐

10. The compound interest on ₹ 15,625 for 9 months at 16% p.a., the interest is compounded quarterly, is :
(a) ₹ 17,576 ☐ (b) ₹ 1,951 ☐ (c) ₹ 18,350 ☐ (d) ₹ 8,765 ☐

Maths Activity

Aim : To find a formula for future value by using compound interest.

Materials required : Chart paper, geometry box, sketch pens.

Process : Suppose you open an account in a bank that pays a guaranteed interest rate, compounded annually. The balance in your account which it will grow to at some point in the future is known as the 'future value' of your starting principal.

For calculating the future value, write P for your starting principal and R for the rate of return expressed as per cent.

Your balance will grow according to the following schedule :

Year	Balance
Now	P
1	$P + \frac{R}{100}P$
2	$\left(P + \frac{R}{100}P\right)\left(1 + \frac{R}{100}\right)$

You can simplify it by noticing that you can keep out factors of $1 + \frac{R}{100}$ from each line. If you do so, the balance comes to a simple pattern :

Year	Balance	Year	Balance
Now	P		
1	$P\left(1 + \frac{R}{100}\right)$	4	$P\left(1 + \frac{R}{100}\right)^4$
2	$P\left(1 + \frac{R}{100}\right)^2$	....	....
3	$P\left(1 + \frac{R}{100}\right)^3$	n	$P\left(1 + \frac{R}{100}\right)^n$

Now, let us take P = ₹ 5000, R = 5%, n = 3 years.

Year	Balance
Now	₹ 5000
1	$5000\left(1 + \frac{5}{100}\right)$ = ₹ 5250
2	$5000\left(1 + \frac{5}{100}\right)^2$ = ₹ 5512.5
3	$5000\left(1 + \frac{5}{100}\right)^3$ = ₹ 5788.125

Chapter 6

Algebraic Expressions And Identities

Sequence of the Chapter

- Types Of Algebraic Expressions
- Degree Of An Algebraic Expression
- Like And Unlike Terms
- Addition And Subtraction Of Algebraic Expressions
- Multiplication Of Algebraic Expressions
- Division Of Algebraic Expressions
- Algebraic Identities (Special Products)

INTRODUCTION

We have learnt a little about algebraic expressions in our previous classes. We have learnt about the terms involved in algebraic expressions like constant, variable, etc. We have also learnt addition and subtraction of algebraic expressions. Let us now recall some of the concepts studied in the previous classes.

- ★ Variables and Constants : One of the basic characteristic of algebra is to use letters or combination of letters to represent numbers. The letters used to represent numbers are called **variables** and the numbers themselves are termed as **constants**. For instance, $a, b, x, y, z, p, q,$ etc. are variables and $2, 9, -6, -3,$ etc. are constants.
- ★ Algebraic Expressions : Algebraic expressions are formed by using variables, constants and operations of addition, subtraction, multiplication and division. A collection of letters (called variables) and real numbers (called constants) that are combined using the operations of addition, subtraction, multiplication and division (except by 0) is called an **algebraic expression**. For instance, $8x$, $2a-4$, $\frac{5}{x^2+2}$, $8a+b$, $4pq+3$ are algebraic expressions.
- ★ Terms : Parts of an algebraic expression which are combined by the '+' or '–' sign, are called its **terms**. For instance, the algebraic expression x^2-4x+4 has three terms : x^2, $-4x$, and 4. The terms x^2 and $-4x$ are the **variable terms** of the expression, and 4 is the **constant term**.
- ★ Coefficients : The numerical factor of a variable term is called the **coefficient** of the variable term. For instance, the coefficient of the variable $-6x$ is -6 and the coefficient of the variable term x^2 is 1.

★ Value of an Algebraic Expression : An algebraic expression takes on a numerical value whenever each variable in the expression is replaced by a real number. For instance, if x is replaced by 4 and y by 8, the algebraic expression $3x + 2y$ becomes $3 \times 4 + 2 \times 8$, which equals 28. We say that $3x + 2y$ has the value 28 when $x = 4$ and $y = 8$.

TYPES OF ALGEBRAIC EXPRESSIONS

An algebraic expression can have one or more than one term. Based upon the number of terms in algebraic expressions, we classify them as follows :

Monomials

An algebraic expression is called a monomial if it contains only one term. *For example,* $3x^2$, $9xy$, $-8a^2$, etc. are all monomials.

Binomials

An algebraic expression is called a binomial if it contains two terms. *For example,* $8x^2 + 10xy$, $ab + \frac{5}{6}xy$, $6p^2 + 2q$, etc. are all binomials.

Trinomials

An algebraic expression is called a trinomial if it contains three terms. *For example,* $3x^2 + 4xy + 2z$, $abc + x^2 + y$, $a^2 - b^2 - c^2$, etc. are all trinomials.

Polynomials

An algebraic expression with more than one term is called a polynomial. Provided that, the exponents of the variables in each term are always non-negative integers. *For example,* $x^2 + 6x + 8$, $a^2 + 2ab + b^2 + 2bc + c^2 + 2ca$, etc. are all polynomials.

DEGREE OF AN ALGEBRAIC EXPRESSION

The degree of each term of an algebraic expression is the sum of the exponents of the variables in that term. The degree of expression is the largest value among the sum.

For example, the degree of expression in one variable $x^3 - 2x^2 + 5x + 6$ is 3, the degree of expression in two variables $x^2y^3 - xy^2 + 8y$ is 5 (*i.e.*, 2 + 3) and the degree of a constant is always zero.

LIKE AND UNLIKE TERMS

Terms having same literal factors are called **like terms**, otherwise they are **unlike terms**.

For example, $5x^2$ and $8x^2$ are like terms because they have same literal factor x^2. But $5xy$ and $2x$ are unlike terms as they have different literal factors (xy and x).

ADDITION AND SUBTRACTION OF ALGEBRAIC EXPRESSIONS

Addition

To add two or more algebraic expressions, we write each expression in a separate row, in such a way that the like terms are aligned in the same column. We then use the procedure of adding like terms.

Example 1 : Add $5a^2 - 3a + 4, - 2a^2 + 6a + 8$ and $6a^2 - 3$.

Solution : We arrange like terms in columns and then add.

$$\begin{array}{rrr} 5a^2 & -\,3a & +\,4 \\ -2a^2 & +\,6a & +\,8 \\ 6a^2 & & -\,3 \\ \hline 9a^2 & +\,3a & +\,9 \\ \hline \end{array}$$

Keep In Mind!

- Unlike terms cannot be grouped to add or subtract.

Vertical method :

$(5a^2 - 3a + 4) + (- 2a^2 + 6a + 8) + (6a^2 - 3)$

$= (5a^2 - 2a^2 + 6a^2) + (- 3a + 6a) + (4 + 8 - 3) = 9a^2 + 3a + 9$

Subtraction

Recall that to subtract 9 from 15, we add 15 and the additive inverse of 9 *i.e.*, -9. Thus $15 - 9 = 15 + (-9) = 6$.

We do the same while subtracting an algebraic expression.

Example 2 : Subtract $(6x^2 - 2x + 4)$ from $(- 5x^2 - 6x + 3)$.

Solution : We arrange the terms so that the term with the highest degree is written first. We write like terms in columns and change the signs of all the terms which are to be subtracted. Then we get :

$$\begin{array}{rrr} -5x^2 & -\,6x & +\,3 \\ 6x^2 & -\,2x & +\,4 \\ (-) & (+) & (-) \\ \hline -11x^2 & -\,4x & -\,1 \\ \hline \end{array}$$

Vertical method :

$(- 5x^2 - 6x + 3) - (6x^2 - 2x + 4)$

$= - 5x^2 - 6x + 3 - 6x^2 + 2x - 4$

$= (- 5x^2 - 6x^2) + (- 6x + 2x) + (3 - 4)$

$= - 11x^2 - 4x - 1$

Exercise : 6 (A)

1. For the following algebraic expressions, identify the terms and their coefficients :

(a) $6xyz^2 - 4xy$ (b) $8 - x + 2x^2$ (c) $3x^2y^2 - 4x^2y^2z^2 + z^2$

(d) $9 - 2ab + 4bc - 2ca$ (e) $\frac{1}{5}x - \frac{1}{2}y + 3xy - x$ (f) $\frac{1}{4}a - \frac{1}{3}b + 1.5\,c$

2. Find the value of the polynomial $8x - 3x^2 + 2$ at :

(a) $x = 0$ (b) $x = 1$ (c) $x = -4$ (d) $x = 3$

3. Classify the following polynomials as monomials, binomials and trinomials :

(a) $x + y$ (b) $3a - 4b$ (c) $1 + 2x - x^3$ (d) $8x - 2y^2 + 8c$

(e) $800x$ (f) $a - 2b^2 + 6a^3$ (g) $9x^2 + 2$ (h) $- 5y^2$

4. Write the degree of the following polynomials :

(a) $5 + 8x^2 - 4x + x^3$ (b) $3xy - 5x^2y + 6x^2y^2$

(c) $5a^5 - 2a^3b + 3ab^3 + 6b^5$ (d) $p^2q^2 - 2pq + 4$

5. Write two like terms for each of the following :

(a) $3xy$ (b) $7x^2y$ (c) $5m^2n$ (d) $6x$

6. Write two unlike terms for each of the following :

(a) $5x^2y$ (b) $6xy^3$ (c) $6ab^2$ (d) $4ab$

7. Add the following expressions :

(a) $2l^2 - 2m^2 + n^2, m^2 - n^2$ and $2n^2 - l^2$

(b) $3x - y + 4xy, y - z + yz$ and $z - 2x + zx$

(c) $ab - bc, bc - ca$ and $ab + bc - ca$

(d) $8xy - 2y^2 + 3zx, 2xy + 4y^2 + zx$ and $-2xy - 3zx$

8. Subtract the following :

(a) $6x^2 - 3y^2 + 4y - 2$ from $6x^2 - 3xy + 9y^2 + 6x - 2y$

(b) $8x - 2y + 4xy - 6$ from $5x^2 + 6x - 3y - 2xy + 4$

(c) $5ab - 2abc - 4ca$ from $8ab - 3abc + 6ca - 3a$

(d) $4xy - 2yz + 8zx$ from $6xy - 3yz + 4zx + 8xyz$

9. What must be added to $x^2 + 6x - 4$ to get $x^3 - 2x^2 + 6x - 3$?

10. What must be subtracted from $x^4 + 3x^2 - 4x + 6$ to get $2x^3 + 5x^2 + x - 4$?

11. Subtract the sum of $-7b + a + c$ and $-2a + 2c + 4b$ from the sum of $6a - 2b + 2c$ and $5c - 4b - a$.

12. The two adjacent sides of a rectangular plot are $-6n + 2o + 8$ and $m - 3n - 2o - 6$. Find the perimeter of the plot.

MULTIPLICATION OF ALGEBRAIC EXPRESSIONS

(i) Rules of signs :

The product of two factors with like signs is positive, while that of unlike signs is negative.

$(+) \times (+) = (+), (+) \times (-) = (-), (-) \times (+) = (-), (-) \times (-) = (+)$

(ii) If x is any variable and m, n are positive integers, then $x^m \times x^n = x^{m+n}$.

For example, $x^3 \times x^6 = x^{3+6} = x^9$, $a^6 \times a^{-2} = a^{6+(-2)} = a^{6-2} = a^4$

The result of two algebraic expressions when multiplied is called **product** and the expressions are called **factors** or **multiplicands**.

For multiplication of algebraic expressions, follow the given rules :

Case 1. For multiplying monomial by monomial

Step I : Multiply all the coefficients.

Step II : Multiply variables and use laws of exponents if variables are same.

Case 2. For multiplying monomial by binomial of the type *a* (*b* + *c*)

Step I : Multiply '*a*' by '*b*' using Case 1 to get the first term of answer.

Step II : Multiply '*a*' by '*c*' to get second term of the answer.

i.e., $a(b + c) = a \times b + a \times c$

Case 3. For multiplying binomial by binomial of the type (*a* + *b*) (*c* + *d*)

Step I : Applying the distributive property, get

$(a + b)(c + d) = a(c + d) + b(c + d)$.

Step II : Use Case 2 to multiply further to get $ac + ad + bc + bd$.

Example 3 : Multiply $3x^3$, $2x^2y$ and $-4xy^2$.

Solution : $3x^3 \times 2x^2y \times (-4xy^2) = \{3 \times 2 \times (-4)\} \times (x^3 \times x^2 \times x) \times (y \times y^2)$

$= (-24) \times (x^{3+2+1}) \times (y^{1+2}) = -24x^6y^3$.

Example 4 : Multiply :

(a) $(6a + 4)$ by $2c$ (b) $(8x^2 + 3y)$ by $-3xy^2$

Solution : (a) $(6a + 4) \times 2c = 2c\,(6a + 4)$

$= 2c \times 6a + 2c \times 4$

$= 12ca + 8c.$

(b) $(8x^2 + 3y) \times (-3xy^2) = -3xy^2\,(8x^2 + 3y)$

$= -3xy^2 \times 8x^2 + (-3xy^2) \times 3y$

$= -24x^3y^2 - 9xy^3.$

Example 5 : Multiply $11x^2$ and $(-x^3 - y^3)$. Verify the result for $x = 1$ and $y = \frac{1}{3}$.

Solution : $(11x^2) \times (-x^3 - y^3) = (11x^2) \times (-x^3) - (11x^2) \times (y^3)$

$= -11 \times (x^2 \times x^3) - 11\,(x^2 \times y^3)$

$= -11x^5 - 11x^2y^3$

Verification : L.H.S. $= 11x^2 \times (-x^3 - y^3)$

$= 11 \times (1)^2 \times \left\{(-1)^3 - \left(\frac{1}{3}\right)^3\right\}$

$= 11 \times \left(-1 - \frac{1}{27}\right) = 11 \times \left(\frac{-27 - 1}{27}\right)$

$= 11 \times \left(\frac{-28}{27}\right) = \frac{-308}{27}$

R.H.S. $= -11x^5 - 11x^2y^3$

$= -11 \times (1)^5 - 11 \times (1)^2 \times \left(\frac{1}{3}\right)^3$

$= -11 - 11 \times \frac{1}{27} = -11 - \frac{11}{27} = \frac{-297 - 11}{27} = \frac{-308}{27}$

$\therefore$ L.H.S. = R.H.S.

Hence verified.

Example 6 : Simplify :

(a) $x^2\,(3x - 3) + 2x^2\,(8x - x^3) - 4x^5$ (b) $5a\,(a - b) - 2b\,(a - c) + c\,(b - c)$

Solution : (a) $x^2\,(3x - 3) + 2x^2\,(8x - x^3) - 4x^5$

$= (x^2 \times 3x) - (x^2 \times 3) + (2x^2 \times 8x) - (2x^2 \times x^3) - 4x^5$

$= 3 \times (x^2 \times x) - 3x^2 + (2 \times 8) \times (x^2 \times x) - 2 \times (x^2 \times x^3) - 4x^5$

$= 3x^3 - 3x^2 + 16x^3 - 2x^5 - 4x^5$

$= 19x^3 - 3x^2 - 6x^5$

$= -6x^5 + 19x^3 - 3x^2.$

(b) $5a\,(a - b) - 2b\,(a - c) + c\,(b - c)$

$= \{(5a \times a) - (5a \times b)\} - \{(2b \times a) - (2b \times c)\} + \{(c \times b) - (c \times c)\}$

$= 5 \times (a \times a) - 5\,(a \times b) - 2 \times (b \times a) + 2 \times (b \times c) + (c \times b) - (c \times c)$

$= 5a^2 - 5ab - 2ab + 2bc + bc - c^2$

$= 5a^2 - 7ab + 3bc - c^2.$

Example 7 : Multiply $(3m^2 - 4mn + 2n^2)$ by $(2m - n)$. Verify the result by taking $m = 3$ and $n = 2$.

Solution : $(3m^2 - 4mn + 2n^2) \times (2m - n)$

Column method :

$$\begin{array}{rl} 3m^2 - 4mn + 2n^2 & \\ 2m - n & \\ \hline 6m^3 - 8m^2n + 4mn^2 \qquad & \text{(Multiply by } 2m\text{)} \\ - 3m^2n + 4mn^2 - 2n^3 & \text{(Multiply by } - n\text{)} \\ \hline 6m^3 - 11m^2n + 8mn^2 - 2n^3 & \text{(Add the terms vertically)} \\ \hline \end{array}$$

Verification : $(3m^2 - 4mn + 2n^2) \times (2m - n) = 6m^3 - 11m^2n + 8mn^2 - 2n^3$

$$\begin{aligned} \text{L.H.S.} &= (3m^2 - 4mn + 2n^2) \times (2m - n) \\ &= (3 \times 3^2 - 4 \times 3 \times 2 + 2 \times 2^2) \times (2 \times 3 - 2) \\ &= (27 - 24 + 8) \times (6 - 2) \\ &= 11 \times 4 = 44 \\ \text{R.H.S.} &= 6m^3 - 11m^2n + 8mn^2 - 2n^3 \\ &= 6 \times 3^3 - 11 \times 3^2 \times 2 + 8 \times 3 \times 2^2 - 2 \times 2^3 \\ &= 6 \times 27 - 11 \times 9 \times 2 + 8 \times 3 \times 4 - 2 \times 8 \\ &= 162 - 198 + 96 - 16 \\ &= (162 + 96) - (198 + 16) \\ &= 258 - 214 = 44 \end{aligned}$$

$\therefore$ L.H.S. = R.H.S.

Hence verified.

Exercise : 6 (B)

1. Multiply the following pairs of monomials :

(a) $8xy, x^3, y^2$ (b) $-3abc, a^2b$ (c) $x^2y^3, -3x$ (d) mn, nl

(e) $xyz^2, xy^2, -3xyz$ (f) $-11x^2z, -y^3, -4z$ (g) $-3xy, \frac{1}{3}xyz^2$ (h) $2ya, -3ab^2c$

2. Find the following products :

(a) $\left(\frac{-x}{4} + 3y\right) \times \frac{-3}{4}x^2y^2$ (b) $x^2y^4 \times \left[\frac{-4}{5}(x^2 - y^2)\right]$

(c) $\frac{1}{4}ab \times \left[2a - \frac{4}{7}b^2a\right]$ (d) $\frac{3}{4}xy\,[x^2 - 2xy + y^2]$

3. Simplify the following :

(a) $\frac{1}{5}x\left(\frac{3}{5}x^2 + \frac{1}{2}y^2\right) - \frac{3}{5}x^2\left(\frac{5}{3}x^2 + 4y^2\right) + 10$

(b) $pq^2\,(q - p^2) + p^2q\,(2p^2 - 4pq) - pq^3\,(1 - 3p)$

(c) $a^2b\,(a^2 - 2b) + 3ab\,(a^3 - 2) + 2a^2b$

(d) $(a + 3)\,(a^2 + 2a - 4) - (a - 2)\,(a^2 - 2a + 4)$

(e) $(x^2 - 3x + 2)\,(2x^2 + 5x - 6) - (3x^2 - 2x + 1)\,(5x - 2)$

4. Find the following products and also find its value for a = 1, b = – 2 :

(a) $(a^2 + 6a - 72) \times (a - 2)$

(b) $18a^2b^2 \times (2a - 3b) \times (3b + 2a)$

5. Simplify the following :

(a) $\left(\frac{1}{5}x - \frac{1}{3}y\right)\left(\frac{3}{5}x + \frac{1}{3}y\right)$

(b) $[8 - (-b)][-(-a) + b]$

(c) $8x(8y - x) + 2x(y + x) + x(x - y)$

(d) $\left(\frac{1}{2}x - y\right)\left(\frac{1}{2}x + y\right)$

6. Find the product of $(x^3 + 2x^2 - 5xy + y)$ and $(x^2 + 8xy + 1)$, and verify the result by taking $x = -1$ and $y = 3$.

7. Add the product of $8p^2q^2$ and $\left(\frac{-1}{4}p^3q\right)$ to the product of $(-6p^3q^2)$ and $4pq$.

8. Subtract $2(a^4 - 3a^2)$ from $2a^2(a^3 - a) - 3a(a^4 + 2a)$.

9. Find the following products and also find its value for x = 1, y = – 1, z = – 2 :

(a) $(x^2 - 5y + 4) \times (5x + 3)$

(b) $\frac{1}{2}(4y^2 + 3z^2) \times (4y^2 - 3z^2)$

(c) $18x^2y^2 \times (2x - 3z) \times (3y - x)$

10. From the product of $(2m + 3n)$ and $(4m - n)$ subtract the product of $(4m + n)$ and $(m + 2n)$.

DIVISION OF ALGEBRAIC EXPRESSIONS

We will now learn how to divide an algebraic expression by a non-zero algebraic expression. Recall that division is the inverse operation of multiplication. Thus, $48 \div 6 = 8$, since $6 \times 8 = 48$.

Similarly $8x^2 \div 2x = 4x$, since $(2x) \times (4x) = 8x^2$.

In this section, we will first understand division of a monomial by another monomial, then explore the division of a polynomial by a monomial and finally understand dividing a polynomial by a polynomial.

Case 1. Division of a monomial by another monomial

Step I : Divide the corresponding coefficients to get the coefficient of answer.

Step II : Divide the variables (use laws of exponents, if variables are same).

Step III : Find the product of the results obtained in steps I and II.

Case 2. Division of a binomial by a monomial of type $(a + b)$ by c

Step I : Write $\frac{a+b}{c}$ as $\frac{a}{c} + \frac{b}{c}$.

Step II : Use Case 1 for finding $\frac{a}{c}$ and $\frac{b}{c}$.

Step III : Add all the quotients to get the result.

Case 3. Division of a polynomial by another polynomial

To divide a polynomial by another polynomial, we use the long division method. The following are the steps to be kept in mind while dividing :

Step I : Arrange both the dividend and the division in descending order.

Step II : Divide the first term of the dividend by the first term of the divisor to estimate the first term of the quotient.

Step III : Multiply all the terms of the divisor by the term of the quotient obtained in step II and then subtract the product from the dividend.

Step IV : Now consider the remainder, if any, in step III, as the new dividend and repeat step II to estimate the next term of the quotient.

Step V : Repeat steps II, III and IV till you obtain a remainder which is either zero or a polynomial of a degree less than that of the divisor.

Example 8 : Divide :

(a) $60x^5y^6$ and $5x^2y^3$

(b) $-32a^4y^7$ by $8a^3y^2$

Solution : (a) $60x^5y^6 \div 5x^2y^3 = \frac{60x^5y^6}{5x^2y^3}$

Here, $\frac{60}{5} = 12, \quad \frac{x^5}{x^2} = x^{5-2} = x^3$

and $\frac{y^6}{y^3} = y^{6-3} = y^3$

$\therefore \quad \frac{60x^5y^6}{5x^2y^3} = 12x^3y^3.$

(b) $-32a^4y^7 \div 8a^3y^2 = \frac{-32a^4y^7}{8a^3y^2}$

Here, $\frac{-32}{8} = -4, \quad \frac{a^4}{a^3} = a^{4-3} = a^1$ or a

and $\frac{y^7}{y^2} = y^{7-2} = y^5$

$\therefore \quad \frac{-32a^4y^7}{8a^3y^2} = -4ay^5.$

Example 9 : Divide :

(a) $(-20x^5y^6 + 30x^2y)$ by $5xy$

(b) $(6a^2b^3 - 10a^4b^5 + 24a^6b^6)$ by $2a^2b$

Solution : (a) Dividing each term of the dividend by the divisor, we get

Quotient $= \frac{-20x^5y^6}{5xy} + \frac{30x^2y}{5xy}$

$= -4x^{5-1}y^{6-1} + 6x^{2-1}y^{1-1}$

$= -4x^4y^5 + 6x^1y^0$

$= -4x^4y^5 + 6x.$ $\quad [\because y^0 = 1]$

(b) $\frac{6a^2b^3 - 10a^4b^5 + 24a^6b^6}{2a^2b}$

$= \frac{6a^2b^3}{2a^2b} - \frac{10a^4b^5}{2a^2b} + \frac{24a^6b^6}{2a^2b}$

$= 3a^{2-2}b^{3-1} - 5a^{4-2}b^{5-1} + 12\,a^{6-2}b^{6-1}$

$= 3a^0b^2 - 5a^2b^4 + 12a^4b^5$

$= 3b^2 - 5a^2b^4 + 12a^4b^5.$ $\quad [\because a^0 = 1]$

Example 10 : Divide $p^6 + 3p^2 + 10$ by $p^3 + 1$ by long division method.

Solution :

$$
\begin{array}{r|l}
p^3+1 & p^6 + 3p^2 + 10 \;\; (p^3-1 \\
 & p^6 + p^3 \\
 & (-) \quad (-) \\
\hline
 & -p^3 + 3p^2 + 10 \\
 & -p^3 \qquad\quad - 1 \\
 & (+) \qquad\quad (+) \\
\hline
 & 3p^2 + 11 \\
\hline
\end{array}
$$

Hence, quotient $= p^3 - 1$ and remainder $= 3p^2 + 11$.

Example 11 : Divide $6a^5 - 27a^3 + 4a^2 + 30a - 9$ by $2a^2 + 1$ and check your answer.

Solution :

$$
\begin{array}{r|l}
2a^2+1 & 6a^5 - 27a^3 + 4a^2 + 30a - 9 \;\; (3a^3-15a+2 \\
 & 6a^5 + 3a^3 \\
 & (-) \quad (-) \\
\hline
 & -30a^3 + 4a^2 + 30a - 9 \\
 & -30a^3 \qquad\quad - 15a \\
 & (+) \qquad\quad (+) \\
\hline
 & 4a^2 + 45a - 9 \\
 & 4a^2 \qquad\quad + 2 \\
 & (-) \qquad\quad (-) \\
\hline
 & 45a - 11 \\
\hline
\end{array}
$$

Dividend $= 6a^5 - 27a^3 + 4a^2 + 30a - 9$,
Divisor $= 2a^2 + 1$, Quotient $= 3a^3 - 15a + 2$, Remainder $= 45a - 11$

Check : Dividend = Divisor × Quotient + Remainder

L.H.S. $= 6a^5 - 27a^3 + 4a^2 + 30a - 9$

R.H.S. $= (2a^2 + 1) \times (3a^3 - 15a + 2) + (45a - 11)$

$= 2a^2 \times (3a^3 - 15a + 2) + 1 \times (3a^3 - 15a + 2) + (45a - 11)$

$= 6a^5 - 30a^3 + 4a^2 + 3a^3 - 15a + 2 + 45a - 11$

$= 6a^5 - 27a^3 + 4a^2 + 30a - 9 =$ L.H.S.

Example 12 : What must be subtracted from $6x^2 - 31x + 47$ so that the resulting polynomial is exactly divisible by $2x - 5$?

Solution :

$$
\begin{array}{r|l}
2x-5 & 6x^2 - 31x + 47 \;\; (3x-8 \\
 & 6x^2 - 15x \\
 & (-) \quad (+) \\
\hline
 & -16x + 47 \\
 & -16x + 40 \\
 & (+) \quad (-) \\
\hline
 & 7 \\
\hline
\end{array}
$$

Dividend = Divisor × Quotient + Remainder

⇒ Dividend – Remainder = Divisor × Quotient

As there is no remainder on R.H.S., it means that

L.H.S. = (Dividend – Remainder) is exactly divisible by the divisor.

Hence, the remainder 7 should be subtracted from the dividend $6x^2 - 31x + 47$ to be exactly divisible by $2x - 5$.

Exercise : 6 (C)

1. Divide the following :

(a) $15a^3b^4c^2$ by $3a^2bc^3$

(b) $25x^9y^9z^6$ by $(-5x^6y^6z^3)$

(c) $(-42abc^3)$ by $(-6a^2bc)$

(d) $(-70p^{10}q^2r^7)$ by $10p^3q^2r^5$

(e) $77mn^3a^2$ by $44\,mna^2$

(f) $49a^2bc^3$ by $(-7abc^2)$

2. Divide the following :

(a) $8x^3 + 6x^2 - 2x$ by $(-2x)$

(b) $4x^3y^2 + 16xy^2 - 8x^2y$ by $4xy$

(c) $a^5 + a^3 + a^2$ by $3a$

(d) $9pq^2 - 18p^3q^4$ by $(-3pq^3)$

(e) $5x^3y^2 + 10x^4y^3 - 25x^5y^4$ by $(-5x^2y^2)$

(f) $9m^2n - 6mn + 12mn^2$ by $\left(-\frac{3}{2}mn\right)$

3. Divide the following :

(a) $(a^3 - 6a^2 + 11a - 6)$ by $(a^2 - 5a + 6)$

(b) $\left(4x^2 + 3x + \frac{1}{2}\right)$ by $(2x + 1)$

(c) $(6y^5 + 4y^4 + 4y^3 + 7y^2 + 27y + 6)$ by $(2y^3 + 1)$

(d) $(8q^3 + 6q^2 + 4q - 1)$ by $(4q + 2)$

4. Divide and check the result :

(a) $(x^4 + 4x^3 - 2x^2 + 10x - 25) \div (x + 5)$

(b) $(10x^3m^2 - 15xm^5 + 5x^2m^7) \div (5xm^2)$

(c) $(5x^3 - 12x^2 + 12x + 13) \div (x^2 - 3x + 4)$

5. What should be subtracted from $4x^4 - 2x^3 - 6x^2 + x - 5$ so that the result is exactly divisible by $2x^2 + x - 2$?

6. Find the values of p and q so that the polynomial $10x^4 + 17x^3 - 62x^2 + px - q$ is exactly divisible by $2x^2 + 7x - 1$.

TRICKY CHALLENGE!

- The area of a rectangular courtyard is $(49x^2 - 35x)$sq. units, and one of its sides measures $7x$ units. What is the measure of the other side?

ALGEBRAIC IDENTITIES (SPECIAL PRODUCTS)

An **algebraic identity** is a statement of equality between two algebraic expressions that is satisfied for all the values of the variables.

For example, $(x - 3)(x + 2) \equiv x^2 - x - 6$ is satisfied for all values of x. The sign '≡' is used to distinguish an identity from an equation.

Standard Identities

I. $(a+b)^2 = a^2 + 2ab + b^2$ or $a^2 + b^2 + 2ab$

In other words, (Sum of the two terms)^2

= (First term)^2 + 2 × (First term) × (Second term) + (Second term)^2

or

= (First term)^2 + (Second term)^2 + 2 × (First term) × (Second term)

Proof : $(a+b)^2 = (a+b)(a+b)$

$= a(a+b) + b(a+b)$

[Distributive property of multiplication over addition]

$= (a \times a + a \times b) + (b \times a + b \times b)$

$= a^2 + ab + ba + b^2$

$= a^2 + ab + ab + b^2$ [Commutative property : $ba = ab$]

$= a^2 + 2ab + b^2$

$\therefore$ $\boxed{(a+b)^2 = a^2 + 2ab + b^2}$

II. $(a-b)^2 = a^2 - 2ab + b^2$ or $a^2 + b^2 - 2ab$

In other words,

(Difference of the two terms)^2

= (First term)^2 + (Second term)^2 – 2 × (First term) × (Second term)

or = (First term)^2 – 2 × (First term) × (Second term) + (Second term)^2

Proof : $(a-b)^2 = (a-b)(a-b)$

$= a(a-b) - b(a-b)$

[Distributive property of multiplication over addition]

$= (a \times a - a \times b) - (b \times a - b \times b)$

$= a^2 - ab - ba + b^2$

$= a^2 - ab - ab + b^2$ [Commutative property : $ba = ab$]

$= a^2 - 2ab + b^2$

$\therefore$ $\boxed{(a-b)^2 = a^2 - 2ab + b^2}$

III. $(a+b)(a-b) = a^2 - b^2$

In other words,

(Sum of the two terms) × (Difference of the two terms) = (First term)^2 – (Second term)^2

Proof : $(a+b)(a-b) = a(a-b) + b(a-b)$

[Distributive property of multiplication over addition]

$= (a \times a - a \times b) + (b \times a - b \times b)$

$= a^2 - ab + ba - b^2$

$= a^2 - ab + ab - b^2$ [Commutative property : $ba = ab$]

$= a^2 - b^2$

$\therefore$ $\boxed{(a+b)(a-b) = a^2 - b^2}$

Example 13 : Find the expansion of the square of $(3x + 2y)$.

Solution : Here, $a = 3x$ and $b = 2y$

So, $a^2 = (3x)^2 = 3 \times 3 \times x \times x = 9x^2$

$b^2 = (2y)^2 = 2 \times 2 \times y \times y = 4y^2$

$2ab = 2 \times 3x \times 2y = 12xy$

Using identity $(a+b)^2 = a^2 + 2ab + b^2$, we get

$(3x+2y)^2 = 9x^2 + 12xy + 4y^2$.

Example 14 : Find :

(a) $(x-4y)^2$ (b) $(6p-4q)^2$ (c) $(4t+2)^2$

Solution (a) Here, $a = x$ and $b = 4y$.

Using identity $(a-b)^2 = a^2 - 2ab + b^2$, we get

$(x-4y)^2 = (x)^2 - 2 \times x \times 4y + (4y)^2$

$= x^2 - 8xy + 16y^2$.

(b) Here, $a = 6p$ and $b = 4q$.

Using identity $(a-b)^2 = a^2 - 2ab + b^2$, we get

$(6p-4q)^2 = (6p)^2 - 2 \times 6p \times 4q + (4q)^2$

$= 36p^2 - 48pq + 16q^2$.

(c) Here, $a = 4t$ and $b = 2$

Using identity $(a+b)^2 = a^2 + 2ab + b^2$, we get

$(4t+2)^2 = (4t)^2 + 2 \times 4t \times 2 + (2)^2$

$= 16t^2 + 16t + 4$.

Example 15 : Find :

(a) $(5x+7)(5x-7)$ (b) $(8a+3b)(8a-3b)$

Solution : (a) Here, $a = 5x$ and $b = 7$

Using identity $(a+b)(a-b) = a^2 - b^2$, we get

$(5x+7)(5x-7) = (5x)^2 - (7)^2$

$= 25x^2 - 49$.

(b) Here, $a = 8a$ and $b = 3b$

Using identity $(a+b)(a-b) = a^2 - b^2$, we get

$(8a+3b)(8a-3b) = (8a)^2 - (3b)^2$

$= 64a^2 - 9b^2$.

Example 16 : Evaluate without actual multiplication :

(a) 98×102 (b) $(105)^2$

Solution : (a) $98 = 100 - 2$

$102 = 100 + 2$

Now, $98 \times 102 = (100-2)(100+2)$

$= (100)^2 - (2)^2$ [Using $(a+b)(a-b) = a^2 - b^2$]

$= 10000 - 4$

$= 9996$.

(b) $(105)^2 = (100+5)^2$

$= (100)^2 + 2 \times 5 \times 100 + (5)^2$ [Using $(a+b)^2 = a^2 + 2ab + b^2$]

$= 10000 + 1000 + 25$

$= 11025$.

Example 17 : If $x + \frac{1}{x} = 11$, find the value of :

(a) $x^2 + \frac{1}{x^2}$ (b) $x^4 + \frac{1}{x^4}$

Solution : (a) $x^2 + \frac{1}{x^2}$

Since $\left(x + \frac{1}{x}\right) = 11 \Rightarrow \left(x + \frac{1}{x}\right)^2 = (11)^2$

$\Rightarrow \quad x^2 + 2 \times x \times \frac{1}{x} + \left(\frac{1}{x}\right)^2 = 11^2$

$\Rightarrow \quad x^2 + 2 + \frac{1}{x^2} = 121$

$\Rightarrow \quad x^2 + \frac{1}{x^2} = 121 - 2 = 119$

$\therefore \quad x^2 + \frac{1}{x^2} = 119.$

(b) $x^4 + \frac{1}{x^4}$

Since $\left(x^2 + \frac{1}{x^2}\right) = 119$ [By part (a)]

Squaring both sides,

$\left(x^2 + \frac{1}{x^2}\right)^2 = (119)^2$

$\Rightarrow \quad (x^2)^2 + \left(\frac{1}{x^2}\right)^2 + 2x^2 \times \frac{1}{x^2} = 14161$

$\Rightarrow \quad x^4 + \frac{1}{x^4} + 2 = 14161$

$\Rightarrow \quad x^4 + \frac{1}{x^4} = 14161 - 2 = 14159$

$\therefore \quad x^4 + \frac{1}{x^4} = 14159.$

Exercise : 6 (D)

1. Expand each of the following by using a suitable identity :

(a) $(x + 4)^2$ (b) $(2a - 8)^2$ (c) $(3p + 3)^2$ (d) $\left(m - \frac{1}{4}\right)^2$

(e) $(p^2 - q^2)^2$ (f) $\left(5a + \frac{1}{2}\right)^2$ (g) $(9a - 8b)^2$ (h) $(6x + 4y)^2$

2. Find the following products using an identity :

(a) $(5p + 6)(5p - 6)$ (b) $(11a - 4b)(11a + 4b)$

(c) $(8a^2 + 3b^2)(8a^2 - 3b^2)$ (d) $\left(\frac{1}{4}x^3 - \frac{1}{5}y^2\right)\left(\frac{1}{4}x^3 + \frac{1}{5}y^2\right)$

3. **Using a suitable identity, evaluate the following :**

(a) $(98)^2$ (b) $(106)^2$ (c) $(208)^2$ (d) $(1004)^2$

(e) 145×202 (f) 96×104 (g) 198×202 (h) 88×92

4. **Using a suitable identity, evaluate the following :**

(a) $\dfrac{92 \times 92 - 5 \times 5}{92 - 5}$ (b) $\dfrac{6.45 \times 6.45 - 0.36 \times 0.36}{41.4729}$

(c) $\dfrac{108 \times 108 - 8 \times 8}{108 + 8}$ (d) $\dfrac{3.2 \times 3.2 - 2(3.2)(1.2) + 1.2 \times 1.2}{3.2 \times 3.2 + 2(3.2)(1.2) + 1.2 \times 1.2}$

5. If $(x - y) = 11$ and $xy = 5$, find the value of $x^2 + y^2$.

6. If $a^2 + 4b^2 = 17$ and $ab = 2$, find $a + 2b$.

7. **If $\left(x + \dfrac{1}{x}\right) = \sqrt{5}$, find the value of :**

(a) $x^2 + \dfrac{1}{x^2}$ (b) $x^4 + \dfrac{1}{x^4}$

8. If $3x + 4y = 9$ and $xy = 2$, find the value of $9x^2 + 16y^2$.

9. **Find the value of x, if :**

(a) $8x = (59)^2 - (51)^2$ (b) $441 - x^2 = (21)^2 - (17)^2$

10. If $\left(x - \dfrac{1}{x}\right)^2 = 49$, find the value of $x^2 + \dfrac{1}{x^2}$.

SUMMARY OF THE CHAPTER

- A collection of letters (called variables) and real numbers (called constants) that are combined using the operations of addition, subtraction, multiplication and division (except by 0) is called an algebraic expression.
- Parts of an algebraic expression which are combined by the '+' or '–' sign are called its terms.
- The numerical factor of a variable term is called the coefficient of the variable term.
- The degree of each term of an algebraic expression is the sum of the exponents of the variables in that term.
- $(a + b)^2 = a^2 + 2ab + b^2$
- $(a - b)^2 = a^2 - 2ab + b^2$
- $a^2 - b^2 = (a + b)(a - b)$

Review Of The Chapter

(Task For Summative Assessment)

1. **Which of the following expressions are polynomials?**

(a) $x^2 + 6$ (b) $2x^5 + x^2 + 1$ (c) $\dfrac{3}{5}x^3 - 2x + 3$

(d) $9x^3$ (e) $5x - 2$ (f) $2x^2\sqrt{x} + 5xy + \sqrt{3}xy$

2. **Write the degree of each of the following polynomials :**

(a) $15x$ (b) $11 - 2x$ (c) $8x - 3x^3 + 4x^2 - 9$ (d) $a^2b^2 - 121$

3. **Simplify :** $\left(\dfrac{1}{3}x^2 - \dfrac{4}{7}x + 11\right) - \left(\dfrac{1}{7}x - 3 + 2x^2\right) - \left(\dfrac{2}{7}x - \dfrac{2}{3}x^2 + 2\right)$.

4. Find the following products :

(a) $\frac{-4}{7}pq^2r^2 \times \frac{-21}{2}pqr$ (b) $\frac{6}{7}a^3b^3c^3 \times \frac{14}{3}a^2b^2c^2 \times \frac{1}{2}ab$

(c) $10pq \times (4p - 9q)$ (d) $6yz \times (x^2y^2 + 2yz)$

5. Find the product of $(8x + 6)$ and $(x^2 - 4x)$ and verify the result for $x = -4$.

6. From the product of $(3l + 4m)$ and $(4l + 5m)$ subtract the product of $(6l + 3m)$ and $(l + 3m)$.

7. Divide :

(a) $x^5 - 8x^4 + 5x^3$ by x^2 (b) $x^4 + 4x^3 + 29x + 20$ by $x + 5$

(c) $9x^2y - 6xy + 12xy^2$ by $\frac{-3}{2}xy$ (d) $-72a^4b^5c^6$ by $-8a^2b^2c^3$

8. Evaluate each of the following using identities :

(a) $(8x + 5)(8x + 5)$ (b) $(3x - 2)(3x - 2)$

(c) $\left(5x^2 + \frac{3}{4}y^2\right)\left(5x^2 - \frac{3}{4}y^2\right)$ (d) $\left(a + \frac{1}{3}\right)\left(a - \frac{1}{3}\right)$

9. Using identities, find the value of :

(a) 102×103 (b) $(78)^2$ (c) 194×206 (d) $(72)^2 - (18)^2$

10. Prove that $2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx = (x - y)^2 + (y - z)^2 + (z - x)^2$.

Multiple Choice Questions (MCQs)

(Task For Formative Assessment)

1. The degree of the polynomial $11x^3 - 2x^2y^2 - 6y^2 + 14$ is :
   (a) 3 ☐ (b) 4 ☐ (c) 5 ☐ (d) none of these ☐
2. The coefficient of a in $-5ab^3$ is :
   (a) 5 ☐ (b) -5 ☐ (c) b^3 ☐ (d) $-5b^3$ ☐
3. Which of these is not a polynomial?
   (a) $x^2 + 5x + 6$ ☐ (b) $2x^3 + \frac{1}{2}x^2 + 4$ ☐ (c) $\frac{2}{x} + \frac{5}{x^2}$ ☐ (d) none of these ☐
4. Which of these is not a monomial?
   (a) $8xyz$ ☐ (b) 5 ☐ (c) $-9y^2 \div 3y$ ☐ (d) $8x^2 + 4$ ☐
5. $(3x - 4y) - (2x - y)$ is equal to :
   (a) $x + 5y$ ☐ (b) $x - 3y$ ☐ (c) $5x - y$ ☐ (d) $x + 3y$ ☐
6. If $x - \frac{1}{x} = 3$, then the value of $x^2 + \frac{1}{x^2}$ is :
   (a) 5 ☐ (b) 7 ☐ (c) 9 ☐ (d) 11 ☐
7. If $xy = 8$ and $x + y = 6$, then the value of $(x^2 + y^2)$ is :
   (a) 18 ☐ (b) 19 ☐ (c) 20 ☐ (d) 49 ☐
8. A rectangular field is half as wide as it is long and is completely enclosed by x metres of fencing. The area in terms of x is :
   (a) $\frac{x^2}{2}$ ☐ (b) $2x^2$ ☐ (c) $\frac{2x^2}{9}$ ☐ (d) $\frac{x^2}{18}$ ☐
9. The value of $x^2 + 2(y^2 + z^2)$ if $x = 2$, $y = 1$ and $z = 3$ is :
   (a) 13 ☐ (b) 20 ☐ (c) 24 ☐ (d) 30 ☐

10. The remainder obtained on dividing $a^3 + 3a^2 - 5a + 4$ by $(a - 1)$ is :
(a) 1 ☐ (b) – 1 ☐ (c) 2 ☐ (d) 3 ☐

Maths Activity

Aim : After performing this activity the students will be able to verify the algebraic identity $a^2 - b^2 = (a + b)(a - b)$ geometrically.

Materials required : Drawing sheet, a pair of scissors, sketch pens, gluestick.

Process :

Step I : Draw and cut a square of side 'a' units. [See fig. (i)] Area of this square is 'a^2'.

Step II : Cut another square with side 'b' such that $b < a$. The area of this square is 'b^2'. [See fig. (ii)]

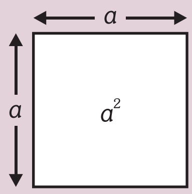


Fig. (i)

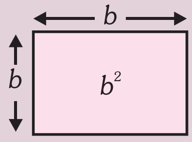


Fig. (ii)

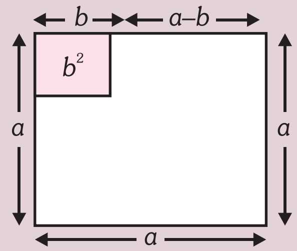


Fig. (iii)

Step III : Paste this small square on the bigger square as shown in fig. (iii).

Step IV : Remove the small square. Area of the remaining portion is $a^2 - b^2$. [See fig. (iv)]

Step V : Join QT. [See fig. (v)]

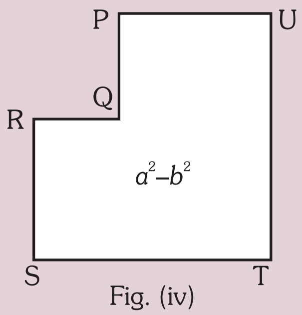


Fig. (iv)

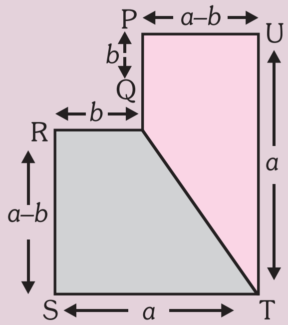
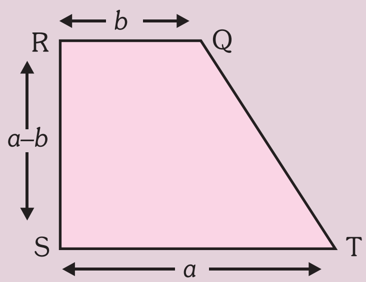


Fig. (v)

Step VI : Colour $RSTQ$ with any colour and $PQTU$ with any different colour.

Step VII : Cut it along QT so as to obtain two trapeziums, $RSTQ$ and $PQTU$ as shown in fig. (vi).

Step VIII : Arrange and paste the two trapeziums as shown in fig. (vii) to form a rectangle $ABCD$.



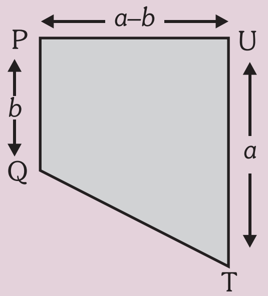


Fig. (vi)

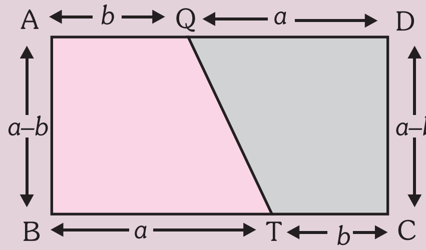


Fig. (vii)

Step IX : Dimensions of this rectangle are $(a + b)$ and $(a - b)$ and its area is $(a + b)(a - b)$.

Observation : We observe that the area of the remaining portion in fig. (iv) is $a^2 - b^2$ which has been further divided into two trapeziums. These two trapeziums when joined together form a rectangle with area $(a + b)(a - b)$. Thus, $a^2 - b^2 = (a + b)(a - b)$

Chapter 7

Factorisation Of Algebraic Expressions

Sequence of the Chapter

- Factorisation
- Factors
- Factorisation Of Algebraic Expressions With A Common Monomial In Each Term
- Factorisation Of Algebraic Expressions With A Common Binomial As Common Factor
- Factorisation Of Algebraic Expressions By Regrouping Terms
- Factorisation Using Identities
- Factorisation Of Quadratic Polynomial In One Variable
- Factorisation Of Quadratic Polynomials Of The Form $ax^2 + bx + c, a \neq 1$

INTRODUCTION

All of us know that, we can multiply several numbers to get a single number, *for example*, the product of $3 \times 8 \times 2 = 48$. We can also find two or more numbers whose product is the given number, *for example*, $105 = 5 \times 3 \times 7$. We have written 105 as the product of three numbers. Here, 5, 3 and 7 are called the **factors** of 105. In the previous chapter, we have learnt how to find the product of two algebraic expressions. Now, in this chapter, we'll learn to find the factors of the given algebraic expressions.

FACTORISATION

Factorisation or factoring is the process by which the factors of a composite number are determined and the number is written as a product of these factors. Just like composite numbers, algebraic expressions also have factors.

FACTORS

When an algebraic expression can be written as the product of two or more algebraic expressions, then each of these expressions is called a factor of the given expression.

For example, $20x^2y^2 = 2 \times 2 \times 5 \times x \times x \times y \times y$

Each 2, 2, 5, x, x, y and y is a factor of $20x^2y^2$. Therefore, $20x^2y^2$ is exactly divisible by 2, 5, x, and y, and hence all are factors of $20x^2y^2$.

Keep In Mind!

- 1 is a factor of every algebraic term.

Highest Common Factor

The highest common factor of two or more monomials is the product of the common factor having greatest numerical coefficient and the common variables with the smallest powers.

Following are the steps involved in finding the HCF of two or more monomials :

Step I : Find the numeric coefficient of each monomial and then find their HCF.

Step II : Find the common variables and their HCF.

Step III : Multiply the HCF in step I and step II to get the final HCF.

Example 1 : Find the HCF of the monomials $6a^3b^2c^4, 10ab^3c$ and $16a^3b^4c^2$.

Solution : $6 = \underline{2} \times 3 \qquad 10 = \underline{2} \times 5 \qquad 16 = \underline{2} \times 2 \times 2 \times 2$

HCF of 6, 10, 16 $= 2$

The common variables in the terms *a*, *b* and *c* are as follows :

Lowest power of *a*, out of $a^2, a, a^3 = a$

Lowest power of *b*, out of $b^2, b^3, b^4 = b^2$

Lowest power of *c*, out of $c^4, c, c^2 = c$

HCF of the variables $= ab^2c$

So, the HCF of $6a^3b^2c^4, 10ab^3c$ and $16a^3b^4c^2 = 2ab^2c$.

FACTORISATION OF ALGEBRAIC EXPRESSIONS WITH A COMMON MONOMIAL IN EACH TERM

Step I : In the algebraic expression, find the highest common factor (HCF) of its terms.

Step II : Express each term of the algebraic expression as the product of the HCF and the quotient, when it is divided by the HCF.

Step III : Using distributive property of multiplication over addition, express the given algebraic expression as the product of its HCF and the quotient obtained in step II.

Example 2 : Factorise :

(a) $8x^2 + 16$ (b) $5x^3y^2 - 4x^2y^4$.

Solution : (a) $8x^2 + 16$

$8x^2 = 2 \times 2 \times 2 \times x \times x$

$16 = \overline{\underline{2 \times 2 \times 2}} \times 2$

HCF of $8x^2$ and $16 = 2 \times 2 \times 2 = 8$

So, we take 8 common from both the terms.

$\therefore \quad 8x^2 + 16 = 8\,(x^2 + 2)$.

(b) $5x^3y^2 - 4x^2y^4$

$5x^3y^2 = 5 \times \underline{x \times x} \times x \times \underline{y \times y}$

$4x^2y^4 = 2 \times \overline{2 \times x \times x} \times \overline{y \times y} \times y \times y$

HCF of $5x^3y^2$ and $4x^2y^4 = x \times x \times y \times y = x^2y^2$

So, we take x^2y^2 common from both the terms.

$\therefore \quad 5x^3y^2 - 4x^2y^4 = x^2y^2\,(5x - 4y^2)$.

Example 3 : Factorise :

(a) $18x^4y^5 + 9x^2y^3 - 24x^4y^3$

(b) $36x^3y^4z + 42x^3y^3z^2 - 54x^2yz^3$

Solution : (a) $18x^4y^5 + 9x^2y^3 - 24x^4y^3$

$18x^4y^5 = 2 \times 3 \times 3 \times x \times x \times x \times x \times y \times y \times y \times y \times y$

$9x^2y^3 = 3 \times 3 \times x \times x \times y \times y \times y$

$24x^4y^3 = 2 \times 2 \times 2 \times 3 \times x \times x \times x \times x \times y \times y \times y$

HCF of $18x^4y^5, 9x^2y^3$ and $24x^2y^3 = 3 \times x \times x \times y \times y \times y = 3x^2y^3$

Taking $3x^2y^3$ common from all the terms, we get $18x^4y^5 + 9x^2y^3 - 24x^4y^3 = 3x^2y^3\ (6x^2y^2 + 3 - 8x^2)$.

(b) $36x^3y^4z + 42x^3y^3z^2 - 54x^2yz^3$

$36x^3y^4z = 2 \times 2 \times 3 \times 3 \times x \times x \times x \times y \times y \times y \times y \times z$

$42x^3y^3z^2 = 2 \times 3 \times 7 \times x \times x \times x \times y \times y \times y \times z \times z$

$54x^2yz^3 = 2 \times 3 \times 3 \times 3 \times x \times x \times y \times z \times z \times z$

HCF of $36x^3y^4z, 42x^3y^3z^2$ and $54x^2yz^3 = 2 \times 3 \times x \times x \times y \times z = 6x^2yz$

Taking $6x^2yz$ common from all the terms, we get

$36x^3y^4z + 42x^3y^3z^2 - 54x^2yz^3 = 6x^2yz\ (6xy^3 + 7xy^2z - 9z^2)$.

FACTORISATION OF ALGEBRAIC EXPRESSIONS WITH A COMMON BINOMIAL AS COMMON FACTOR

Step I : Locate the common binomial.

Step II : Write the given algebraic expression as a product of this common binomial and the quotient obtained on dividing the given algebraic expression by this binomial.

Example 4 : Factorise the following :

(a) $6\ (2x - 5) + 5\ (2x - 5)$ (b) $2m\ (x + 3y) + 3n\ (x + 3y)$

(c) $9\ (4x - 7y)^2 - 15\ (4x - 7y)$

Solution : (a) $6\ (2x - 5) + 5\ (2x - 5) = (6 + 5)\ (2x - 5)$ [Taking $(2x - 5)$ common]

$= 11\ (2x - 5)$.

(b) $2m\ (x + 3y) + 3n\ (x + 3y) = (2m + 3n)\ (x + 3y)$.

[Taking $(x + 3y)$ common]

(c) $9\ (4x - 7y)^2 - 15\ (4x - 7y)$

$= 3 \times 3 \times (4x - 7y) \times (4x - 7y) - 3 \times 5 \times (4x - 7y)$

$= 3\ (4x - 7y)\ [3\ (4x - 7y) - 5]$

[Taking $3\ (4x - 7y)$ common]

$= 3\ (4x - 7y)\ (12x - 21y - 5)$.

Example 5 : Factorise the following :

(a) $(3a - b)x + (b - 3a)y$ (b) $8\ (x - y)^2 - 16\ (y - x)$

(c) $(5x - 4y)^2 - 10x + 8y$ (d) $4x + 6y - 2\ (2x + 3y)^2$

Solution : (a) $(3a - b)x + (b - 3a)y = (3a - b)x - (3a - b)y$

[Taking (-1) common from $(b - 3a)$]

$= (3a - b)\ (x - y)$. [Taking $(3a - b)$ common]

(b) $8\ (x - y)^2 - 16\ (y - x) = 8\ (x - y)^2 + 16\ (x - y)$

[Taking (-1) common from $(y - x)$]

$= 2 \times 2 \times 2 \times (x - y)^2 + 2 \times 2 \times 2 \times 2\ (x - y)$

$= 2 \times 2 \times 2 \times (x - y)\,[(x - y) + 2]$

$= 8\,(x - y)\,(x - y + 2).$ [Taking $8\,(x - y)$ common]

(c) $(5x - 4y)^2 - 10x + 8y = (5x - 4y)^2 - 2 \times 5x + 2 \times 4y$

$= (5x - 4y)^2 - 2\,(5x - 4y)$

[Taking (-2) common from $(-10x + 8y)$]

$= (5x - 4y)\,(5x - 4y - 2).$

[Taking $(5x - 4y)$ common]

(d) $4x + 6y - 2\,(2x + 3y)^2 = 2 \times 2x + 2 \times 3y - 2\,(2x + 3y)^2$

$= 2\,(2x + 3y) - 2\,(2x + 3y)^2$ [Taking 2 common from $4x + 6y$]

$= 2\,(2x + 3y)\,[1 - (2x + 3y)]$ [Taking $2\,(2x + 3y)$ common]

$= 2\,(2x + 3y)\,(1 - 2x - 3y).$

Exercise : 7 (A)

1. Find the HCF of the following monomials :

(a) $4x^5$ and $16x^7$ (b) $3ab^2c$ and $27a^2b$ (c) $50x^5y^2$, $24x^4y^3$ and $28x^5y$

(d) $-3x^3y^3z^2$, $15x^2yz$ and $-21x^2y^2z^3$

(e) $-7x^5y^6z^7$, $14x^6y^5z^4$, $-35x^6y^4z^3$ and $42x^8y^5z^5$

2. Factorise the following :

(a) $5x - 45$ (b) $3a^3b - 18ab^2$ (c) $-5x^2y + 10x^3y^2 + 5x^3y$

(d) $ab + 3ac + a^2$ (e) $48x^4y^3z^5 - 36x^2y^3z^4$ (f) $ax^2y + bxy^2 + cxyz$

(g) $25x^3y^3z^4 - 50x^5z^6 + 95x^4y^2z^3$ (h) $18x^6 + 20x^5 - 12x^3$

3. Factorise the following :

(a) $5x\,(4x - 7y) - 15x^3\,(4x - 7y)$ (b) $3\,(a - 2b)^2 - 3\,(a - 2b)$

(c) $x\,(5 - m) + y\,(m - 5)$ (d) $p\,(x + y) + 2q\,(x + y)^3 + r\,(x + y)$

(e) $(2x + 3y)\,(6x - 7y) - (2x + 3y)\,(7y - 6x)$ (f) $(5x - 6y)^2 - 15x + 18y$

(g) $18\,(x - 2y)^3 - 40\,(x - 2y)^2$

(h) $14x\,(6x - 4y) - 12x^2\,(6x - 4y) + 10x\,(4y - 6x)$

FACTORISATION OF ALGEBRAIC EXPRESSIONS BY REGROUPING TERMS

When a polynomial has more than three terms, it can sometimes be factored by a method called **factorisation by regrouping terms**. *For example,* to factorise $ax + ay + bx + by$, we do the following :

Step I : Collect the terms into two groups so that each has a common factor.

$ax + ay + bx + by = (ax + ay) + (bx + by)$

common factors

Step II : Taking the common factors out of the two groups, we get

$ax + ay + bx + by = a\,(x + y) + b\,(x + y)$

Step III : The expression $(x + y)$ is common in both the groups, which can be further factored out to obtain :

$ax + ay + bx + by = (x + y)\,(a + b).$

Example 6 : Factorise by grouping the terms :

(a) $x^2 + xy + 3x + 3y$ (b) $10ab + 10 + 4b + 25a$

Solution : (a) $x^2 + xy + 3x + 3y$

Step I : There is no factor common to all the terms. There are four terms in the given expression. Pair two terms in such a way that they have a common factor.

Step II : Consider the first two terms :

$x^2 + xy = x(x + y)$

Step III : Now take the last two terms :

$3x + 3y = 3(x + y)$

Note that $(x + y)$ is a common factor in step II and step III.

Step IV : Combine step II and step III together.

$(x^2 + xy) + (3x + 3y) = x(x + y) + 3(x + y) = (x + 3)(x + y)$.

(b) $10ab + 10 + 4b + 25a$

Step I : There is no factor common to all the terms. There are four terms in the given expression. On combining first and second terms or third and fourth terms, there is no common factor. So terms need to be regrouped.

Step II : Pair the first and third term :

$10ab + 4b = 2 \times 5ab + 2 \times 2b = 2b(5a + 2)$

Step III : Pair the second and fourth term :

$10 + 25a = 2 \times 5 + 5 \times 5a = 5(2 + 5a)$

$= 5(5a + 2)$

Note that $(5a + 2)$ is a common factor in step II and step III.

Step IV : Combine step II and step III together.

$(10ab + 4b) + (10 + 25a) = 2b(5a + 2) + 5(5a + 2)$

$= (2b + 5)(5a + 2)$.

Example 7 : Factorise the following :

(a) $pq + rq - px - rx$ (b) $a^2 + bc + ab + ac$

Solution : (a) $pq + rq - px - rx = (pq - px) + (rq - rx)$ [Regrouping]

$= p(q - x) + r(q - x)$ [Taking p and r common]

$= (p + r)(q - x)$. [Taking $(q - x)$ common]

(b) $a^2 + bc + ab + ac = (a^2 + ab) + (bc + ac)$ [Regrouping]

$= a(a + b) + c(a + b)$ [Taking a and c common]

$= (a + c)(a + b)$. [Taking $(a + b)$ common]

Example 8 : Factorise the following :

(a) $8xy - y^2 + 16xz - 2yz$

(b) $l^3x + l^2(x - y) - l(y + z) - z$

Solution : (a) $8xy - y^2 + 16xz - 2yz = (8xy - y^2) + (16xz - 2yz)$ [Regrouping]

$= y(8x - y) + 2z(8x - y)$

$= (y + 2z)(8x - y)$. [Taking $(8x - y)$ common]

(b) $l^3x + l^2(x - y) - l(y + z) - z = l^3x + l^2x - l^2y - ly - lz - z$ [Simplifying the expression]

$= (l^3x + l^2x) - (l^2y + ly) - (lz + z)$ [Regrouping]

$= l^3x(l + 1) - ly(l + 1) - z(l + 1)$

$= (l^2x - ly - z)(l + 1).$ [Taking $(l + 1)$ common]

Exercise : 7 (B)

1. Factorise the following :

(a) $a^2 + ab + 9a + 9b$

(b) $x - 5y^2z + 5xy - yz$

(c) $xy - mn + nx - my$

(d) $8xyz + 16xy + 2z + 4$

(e) $p^2 - p(x + 2y) + 2xy$

(f) $15ab + 15 + 9b + 25a$

(g) $px - 2py - qx + 2qy$

(h) $ar + br + at + bt$

FACTORISATION USING IDENTITIES

Factorisation Of Binomial Expressions When Expressed As The Difference Of Two Squares

When the given algebraic expression is the difference of squares of two terms, then it can be factorised as the product of the sum and the difference of the two terms.

$$a^2 - b^2 = (a + b)(a - b)$$

Example 9 : Factorise : (a) $36x^2 - 49y^2$ (b) $16x^2 - (p + q)^2$

Solution : (a) $36x^2 - 49y^2 = (6x)^2 - (7y)^2$

$= (6x + 7y)(6x - 7y).$ [Using $a^2 - b^2 = (a + b)(a - b)$]

(b) $16x^2 + (p + q)^2 = (4x)^2 - (p + q)^2$

$= (4x + p + q)\{4x - (p + q)\}$ [Using $a^2 - b^2 = (a + b)(a - b)$]

$= (4x + p + q)(4x - p - q).$

Example 10 : Factorise :

(a) $64x^4 - 25$ (b) $p^4 - (q + r)^4$

Solution : (a) $64x^4 - 25 = (8x^2)^2 - (5)^2$

$= (8x^2 + 5)(8x^2 - 5).$ [Using $a^2 - b^2 = (a + b)(a - b)$]

(b) $p^4 - (q + r)^4 = (p^2)^2 - \{(q + r)^2\}^2$

$= \{p^2 + (q + r)^2\}\{p^2 - (q + r)^2\}$ [Using $a^2 - b^2 = (a + b)(a - b)$]

$= \{p^2 + (q + r)^2\}(p + q + r)\{p - (q + r)\}$ [Using $a^2 - b^2 = (a + b)(a - b)$]

$= \{p^2 + (q + r)^2\}(p + q + r)(p - q - r).$

Example 11 : Factorise :

(a) $81(x + y)^2 - 121(a + b)^2$ (b) $x^{12}y^4 - x^4y^{12}$

(c) $9p^2q - \dfrac{q}{16p^2}$ (d) $a(a + c) - b(b + c)$

Solution : (a) $81(x+y)^2 - 121(a+b)^2 = \{9(x+y)\}^2 - \{11(a+b)\}^2$

$= \{9(x+y) + 11(a+b)\}\{9(x+y) - 11(a+b)\}$ [Using $a^2 - b^2 = (a+b)(a-b)$]

$= (9x + 9y + 11a + 11b)(9x + 9y - 11a - 11b)$.

(b) $x^{12}y^4 - x^4y^{12} = (x^6)^2(y^2)^2 - (x^2)^2(y^6)^2$

$= (x^6y^2)^2 - (x^2y^6)^2$

$= (x^6y^2 + x^2y^6)(x^6y^2 - x^2y^6)$ [Using $a^2 - b^2 = (a+b)(a-b)$]

$= (x^6y^2 + x^2y^6)\{(x^3y)^2 - (xy^3)^2\}$

$= (x^6y^2 + x^2y^6)(x^3y + xy^3)(x^3y - xy^3)$ [Using $a^2 - b^2 = (a+b)(a-b)$]

$= x^2y^2(x^4 + y^4)\,xy(x^2 + y^2)\,xy(x^2 - y^2)$

$= x^4y^4(x^4 + y^4)(x^2 + y^2)(x^2 - y^2)$

$= x^4y^4(x^4 + y^4)(x^2 + y^2)(x + y)(x - y)$.

[Using $a^2 - b^2 = (a+b)(a-b)$]

Alternate method :

$x^{12}y^4 - x^4y^{12} = x^4y^4(x^8 - y^8) = x^4y^4\{(x^4)^2 - (y^4)^2\}$

$= x^4y^4(x^4 + y^4)(x^4 - y^4)$ [Using $a^2 - b^2 = (a+b)(a-b)$]

$= x^4y^4(x^4 + y^4)\{(x^2)^2 - (y^2)^2\}$

$= x^4y^4(x^4 + y^4)(x^2 + y^2)(x^2 - y^2)$

[Using $a^2 - b^2 = (a+b)(a-b)$]

$= x^4y^4(x^4 + y^4)(x^2 + y^2)(x + y)(x - y)$.

[Using $a^2 - b^2 = (a+b)(a-b)$]

(c) $9p^2q - \dfrac{q}{16p^2} = q\left(9p^2 - \dfrac{1}{16p^2}\right) = q\left\{(3p)^2 - \left(\dfrac{1}{4p}\right)^2\right\}$

$= q\left(3p + \dfrac{1}{4p}\right)\left(3p - \dfrac{1}{4p}\right)$. [Using $a^2 - b^2 = (a+b)(a-b)$]

(d) $a(a+c) - b(b+c) = a^2 + ac - b^2 - bc$ [Simplifying the expression]

$= (a^2 - b^2) + (ac - bc)$ [Regrouping]

$= (a+b)(a-b) + c(a-b)$

[Using $a^2 - b^2 = (a+b)(a-b)$]

$= (a + b + c)(a - b)$.

Example 12 : Evaluate $(102)^2 - (98)^2$ using the identity $a^2 - b^2 = (a+b)(a-b)$.

Solution : $(102)^2 - (98)^2$, Here $a = 102$, $b = 98$

$\therefore\ (102)^2 - (98)^2 = (102 + 98)(102 - 98) = 200 \times 4 = 800$.

Exercise : 7 (C)

1. Factorise the following :

(a) $x^2 - 25$ (b) $16x^2y^2 - 25$ (c) $(a+b)^2 - (c+d)^2$

(d) $63x^2 - 112y^2$ (e) $10^4 - 81q^4$ (f) $16(2x-1)^2 - 36y^2$

(g) $x^3 - 16x$ (h) $144 - 81y^2$ (i) $m^8 - n^8$

(j) $(5x - 4y)^2 - 16z^2$ (k) $25x^4 - (z - x)^4$ (l) $ab^9 - ba^9$

2. Evaluate using $a^2 - b^2 = (a + b)(a - b)$, where :

(a) $a = 9, b = 4$ (b) $a = 6.4, b = 3.6$ (c) $a = 62, b = 58$

Factorisation Of Algebraic Expressions Expressible As A Perfect Square

When the given algebraic expression is the sum of the squares of two terms with twice the product of the terms added or subtracted, then it is factorised by using following identities :

$$a^2 + 2ab + b^2 = (a + b)^2 = (a + b)(a + b)$$
$$a^2 - 2ab + b^2 = (a - b)^2 = (a - b)(a - b)$$

Example 13 : Factorise :

(a) $16x^2 + 40xy + 25y^2$ (b) $9x^2 - 12x + 4$

Solution : (a) $16x^2 + 40xy + 25y^2 = (4x)^2 + 2(4x)(5y) + (5y)^2$

$= (4x + 5y)^2$ [Using $a^2 + 2ab + b^2 = (a + b)^2$]

$= (4x + 5y)(4x + 5y)$.

(b) $9x^2 - 12x + 4 = (3x)^2 - 2(3x)(2) + (2)^2$

$= (3x - 2)^2$ [Using $a^2 - 2ab + b^2 = (a - b)^2$]

$= (3x - 2)(3x - 2)$.

Example 14 : Factorise :

(a) $(x + y)^2 + 2(x + y) + 1$ (b) $32x^2 + 48x + 18$

(c) $x^4 - (x - y)^4$ (d) $x^2y^2 - 2(xy + yz - xz)$

Solution : (a) $(x + y)^2 - 2(x + y) + 1 = (x + y)^2 + 2(x + y)(1) + (1)^2$

$= (x + y + 1)^2$ [Using $a^2 + 2ab + b^2 = (a + b)^2$]

$= (x + y + 1)(x + y + 1)$.

(b) $32x^2 + 48x + 18 = 2(16x^2 + 24x + 9)$

$= 2\{(4x)^2 + 2(4x)(3) + (3)^2\}$

$= 2(4x + 3)^2$ [Using $a^2 + 2ab + b^2 = (a + b)^2$]

$= 2(4x + 3)(4x + 3)$.

(c) $x^4 - (x - y)^4 = (x^2)^2 - \{(x - y)^2\}^2$

$= \{x^2 + (x - y)^2\}\{x^2 - (x - y)^2\}$ [Using $a^2 - b^2 = (a + b)(a - b)$]

$= (x^2 + x^2 - 2xy + y^2)\{x^2 - (x^2 - 2xy + y^2)\}$

[Using $(a - b)^2 = a^2 - 2ab + b^2$]

$= (2x^2 - 2xy + y^2)(x^2 - x^2 + 2xy - y^2)$

$= (2x^2 - 2xy + y^2)(2xy - y^2)$

$= (2x^2 - 2xy + y^2)(2x - y)y$.

Alternate method :

$x^4 - (x - y)^4 = (x^2)^2 - \{(x - y)^2\}^2$

$= \{x^2 + (x - y)^2\}\{x^2 - (x - y)^2\}$

[Using $a^2 - b^2 = (a + b)(a - b)$]

$= (x^2 + x^2 - 2xy + y^2)(x + x - y)\{x - (x - y)\}$

[Using $(a - b)^2 = a^2 - 2ab + b^2$ and $a^2 - b^2 = (a + b)(a - b)$]

$= (2x^2 - 2xy + y^2)(2x - y)(x - x + y)$

$= (2x^2 - 2xy + y^2)(2x - y)y$.

(d) $x^2 + y^2 - 2(xy + yz - xz) = x^2 + y^2 - 2xy - 2yz + 2xz$

$= (x^2 - 2xy + y^2) + (2xz - 2yz)$ [Regrouping]

$= (x - y)^2 + 2z\,(x - y)$ [Using $a^2 - 2ab + b^2 = (a - b)^2$]

$= (x - y + 2z)\,(x - y).$ [Taking $(x - y)$ common from both the terms]

Exercise : 7 (D)

1. Factorise the following :

(a) $1 + 2a + a^2$ (b) $x^2 - x + \frac{1}{4}$ (c) $m^6 - 4m^3 + 4$

(d) $36x^2 - 36x + 9$ (e) $(l + m)^2 - 4lm$ (f) $x^2 + 14x + 49$

(g) $a^{16} - b^{16} + a^8 + b^8$ (h) $y^2 + 20y + 100$ (i) $x^4 - 8x^2y^2 + 16y^2 - 289$

(j) $p^2 + 2p^2q^2 + q^4$ (k) $49x^2 - 70xy + 25y^2$ (l) $p^4 - (p + q)^4$

FACTORISATION OF QUADRATIC POLYNOMIAL IN ONE VARIABLE

Step I : In the quadratic polynomial $x^2 + ax + b$, find a (coefficient of x) and b (constant term).

Step II : Find two numbers p and q such that $p + q = a$ and $pq = b$.

Step III : Split the middle term as the sum of px and qx, *i.e.*, $x^2 + (p + q)\,x + b$

$\Rightarrow \quad x^2 + px + qx + b.$

Step IV : Factorise the algebraic expression by grouping.

Consider the quadratic polynomials : $x^2 + 5x + 6$, $a^2 - 4a - 12$.

We know that,

$$x^2 + (a + b)x + ab = (x + a)\,(x + b)$$

$$x^2 + (a - b)x - ab = (x + a)\,(x - b)$$

or

$$x^2 + (b - a)x - ab = (x - a)\,(x + b)$$

and, $$x^2 - (a + b)x + ab = (x - a)\,(x - b)$$

We can factorise these polynomials as follows :

$x^2 + 5x + 6$, Here $a + b = 5$, $ab = 6 \Rightarrow a = 2$, $b = 3$

$\therefore \quad x^2 + 5x + 6 = x^2 + (2 + 3)\,x + 6$

$= x^2 + 2x + 3x + 6$

$= x\,(x + 2) + 3\,(x + 2) = (x + 2)\,(x + 3).$

$a^2 - 4a - 12$, Here $a + b = -4$, $ab = -12 \Rightarrow a = -6$, $b = 2$

$\therefore \quad a^2 - 4a - 12 = a^2 + (-6 + 2)\,a - 12$

$= a^2 + (2 - 6)\,a - 12 = a^2 + 2a - 6a - 12$

$= a\,(a + 2) - 6\,(a + 2) = (a + 2)\,(a - 6).$

Example 15 : Factorise :

(a) $x^2 + 7x + 10$ (b) $x^2 + 2x - 3$

Solution : (a) $x^2 + 7x + 10$

To factorise this expression, we need to find p and q such that $p + q = 7$, $pq = 10$.

Factors of 10 are 2×5, $(-2) \times (-5)$, 10×1, $(-10) \times (-1)$. Out of these factors, we take only 2×5 because $2 + 5 = 7$.

$\therefore \quad p = 2$ and $q = 5$

$\therefore \quad x^2 + 7x + 10 = x^2 + (2+5)x + 10 = x^2 + 2x + 5x + 10$

$= x(x+2) + 5(x+2)$

$= (x+5)(x+2)$. [Taking $(x+2)$ common]

(b) $x^2 + 2x - 3$

To factorise this expression, we need to find p and q such that $p + q = 2$, $pq = -3$.

So, we take $p = 3$ and $q = -1$ as $pq = 3 \times (-1) = -3$ and $p + q = 3 - 1 = 2$.

$\therefore \quad x^2 + 2x - 3 = x^2 + (3-1)x - 3$

$= x^2 + 3x - x - 3$

$= x(x+3) - 1(x+3)$

$= (x-1)(x+3)$. [Taking $(x+3)$ common]

FACTORISATION OF QUADRATIC POLYNOMIALS OF THE FORM $ax^2 + bx + c$, $a \neq 1$

Step I : In the quadratic polynomial $ax^2 + bx + c$, locate a (coefficient of x^2), b (coefficient of x) and c (constant term).

Step II : Find the product ac, *i.e.*, product of coefficient of x^2 and the constant term.

Step III : Split the coefficient of x, *i.e.*, b in two parts, such that Ist part + IInd part = b and Ist part × IInd part = ac.

Step IV : Factorise the algebraic expression in step III by grouping the terms.

Example 16 : Factorise :

(a) $10x^2 - 83x - 17$ (b) $(a+2b)^2 - 13(a+2b) + 36$

Solution : (a) $10x^2 - 83x - 17$

coefficient of $x^2 = 10$, coefficient of $x = -83$, constant term $= -17$

Split the middle term, *i.e.*, -83 in such a way that the sum of the two parts $= -83$ and their product $= 10 \times -17 = -170$.

So, we take -85 and 2 because $(-85) + 2 = -83$ and $-85 \times 2 = -170$.

$\therefore \quad 10x^2 - 83x - 17 = 10x^2 + (-85 + 2)x - 17$

$= 10x^2 - 85x + 2x - 17$

$= 5x(2x - 17) + 1(2x - 17)$

$= (5x + 1)(2x - 17)$.[Taking $(2x - 17)$ common]

(b) $(a+2b)^2 - 13(a+2b) + 36$

Put $a + 2b = p$.

So, the given expression becomes $p^2 - 13p + 36$.

coefficient of $p^2 = 1$, coefficient of $p = -13$, constant term $= 36$

Split the middle term, *i.e.*, -13 in such a way that the sum of the two parts $= -13$ and their product $= 36$.

So, we take -9 and -4 because $-9 + (-4) = -13$ and $(-9) \times (-4) = 36$.

$\therefore \quad p^2 - 13p + 36 = p^2 + (-9-4)p + 36$

$= p^2 - 9p - 4p + 36$

$= p\,(p - 9) - 4\,(p - 9)$

$= (p - 4)\,(p - 9)$ [Taking $(p - 9)$ common]

Putting the value of p,

$= (a + 2b - 4)\,(a + 2b - 9)$.

Exercise : 7 (E)

1. Factorise the following algebraic expressions by splitting the middle term :

(a) $a^2 + 5a - 50$ (b) $x^2 - 9x + 14$ (c) $x^3 + 2x^2 - 8x$

(d) $2p^2 - 17p - 30$ (e) $m^2 + 16m + 60$ (f) $x^2 - x - 12$

(g) $x^2 + 13x + 30$ (h) $6x^2 - 17xy + 12y^2$ (i) $x^2 - 11x - 42$

(j) $x^2 - 14x - 51$ (k) $p^2 - 19x + 84$ (l) $\frac{1}{3}x^2 - 2x - 9$

SUMMARY OF THE CHAPTER

- Factorisation is the process by which the factors of a composite number are determined and the number is written as a product of these factors.
- The highest common factor of two or more monomials is the product of the common factor having greatest numerical coefficient and the common variables with the smallest powers.

Review Of The Chapter

(Task For Summative Assessment)

1. Factorise :

(a) $ax^2y + bxy^2 + cxyz$ (b) $x^2 - xz + xy - yz$ (c) $(lx - my)^2 + (mx + ly)^2$

(d) $36p^5q^4 + 72p^4q^5$ (e) $2a^3b - 4a^2$ (f) $z - 6 + 6xy - xyz$

2. Factorise :

(a) $63x^2 - 112y^2$ (b) $l^4 - (m + n)^4$ (c) $x^2 - x + \frac{1}{4}$

(d) $m^2 - (n + p)^2$ (e) $a\,(a + c) - b\,(b + c)$ (f) $256a^8 - b^8$

3. Factorise :

(a) $x^2 + 2x + 1$ (b) $a^4 \div b^4$ (c) $x^4 + 22x^2 + 121$

(d) $x^8 - y^8 + x^4 - y^4$ (e) $x^2 - y^2 - 9z^2 + 6yz$ (f) $49a^6 - 28a^3b^3 + 4b^6$

4. Factorise :

(a) $x^2 + 6x + 8$ (b) $a^2 + 5a - 104$ (c) $x^2 - 9x + 20$

(d) $(x + 2)^2 - 8\,(x + 2) + 16$ (e) $x^2 - x - 56$ (f) $3\,(2p - q)^2 + 14\,(2p - q) + 8$

Multiple Choice Questions (MCQs)

(Task For Formative Assessment)

1. The HCF of $3xy^2z$ and $-6x^2y$ is :

(a) $3xy$ ☐ (b) $-3xz$ ☐ (c) $-2x^2yz$ ☐ (d) $-6x^2yz$ ☐

2. The factorisation of $a^4 - b^4$ is :

(a) $a^2 - b^2$ ☐ (b) $(a - b)\,(a^3 - b^3)$ ☐

(c) $(a - b)\,(a + b)\,(a^2 + b^2)$ ☐ (d) $(a - b)\,(a + b)\,(a^2 - b^2)$ ☐

3. The value of $61^2 - 59^2$ is :

(a) 120 (b) 240 (c) 360 (d) 380

4. $(x - y)^2 - z^2$ is equal to :

(a) $(x - y - z)(x - y - z)$ (b) $(x - y + z)(x - y - z)$

(c) $(x - y + z)(x + y + z)$ (d) $(x + y + z)(z + y + z)$

5. Factors of $a^2 - b^2 + 2bc - c^2$ are :

(a) $(a + b + c)$ and $(a - b + c)$ (b) $(a + b - c)$ and $(a + b + c)$

(c) $(a - b + c)$ and $(a + b - c)$ (d) $(a + b + c)$ and $(a - b - c)$

6. Factors of $xy - pq + xq - py$ are :

(a) $(x - y)(p - q)$ (b) $(x - p)(y + q)$

(c) $(x - q)(p + y)$ (d) $(p - x)(q - y)$

7. Which one of the following is not a factor of $x^3 + 2x^2 + x$?

(a) $(x + 2)$ (b) $(x + 1)$ (c) x (d) $x(x + 1)$

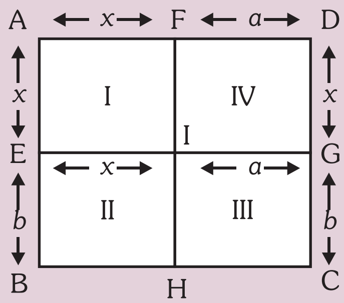
Maths Activity

Aim : To verify the algebraic identity :

$$(x + a)(x + b) = x^2 + (a + b)x + ab$$

Materials required : Drawing sheet, scale, pencil, a pair of scissors, gluestick, marker pen.

Process :



Step I : Draw a rectangle *ABCD* on the drawing sheet with sides $(x + a)$ and $(x + b)$.

Step II : Mark a point *E* on *AB* and *F* on *AD* so that $AE = x = AF$. Now $AB = x + b$, $AD = x + a$, $AE = x$, and $EB = b$, $AF = x$, $FD = a$

.

Step III : From *E* draw *EG* parallel to *AD* meeting *DC* at *G*. From *F* draw *FH* parallel to *AB* meeting *BC* at *H* and intersecting *EG* at *I*.

Step IV : We obtain four parts of rectangle *ABCD*.

Step V : Four parts are a square I (*AEIF*) whose area is x^2, rectangle II (EBHI) whose area is bx, rectangle III (*IHCG*) whose area is ab, rectangle IV (*FIGD*) whose area is ax.

Observation : Area of the rectangle $ABCD = (x + a)(x + b)$.

Also, area of the rectangle *ABCD* = area of the square I + area of the rectangle II + area of the rectangle III + area of the rectangle IV.

$= x^2 + bx + ab + ax = x^2 + ax + bx + ab = x^2 + (a + b)x + ab$

Thus, $(x + a)(x + b) = x^2 + (a + b)x + ab$

Chapter 8

Linear Equations

Sequence of the Chapter

- Linear Equation
- Rules For Solving A Linear Equation
- Solving Equations Having Variable Terms On One Side And Number(s) On The Other Side
- Solving Equations Having Variable Terms And Number(s) On Both Sides
- Solving Linear Equations By Cross-Multiplication Method
- Applications Of Linear Equations To Word Problems

INTRODUCTION

We are now well-known with algebraic expressions and their operations. Any algebraic expression becomes an equation when it contains an 'equal to (=)' sign. In other words, an equation is a statement of equality involving one or more unknown measures or variables.

For example, $3x + 4 = 10$, $3x^2 + 2x - 1 = 0$, etc. are equations.

LINEAR EQUATION

A **linear equation** is an equation involving expressions of degree one. *For example,* $y = 5$, $x = 3x + 2$ are linear equations, but $a^2 = 4 + a$ or $\frac{1}{x} = \frac{2}{y}$ are not.

Linear Equation In One Variable

A linear equation, having only one variable, is called a linear equation in one variable. *For example,* $5x - 2 = 8$, $\frac{2x-3}{4} + \frac{x-1}{5} = 6$ are linear equations in one variable.

Solution Of A Linear Equation

The value of a variable, which satisfies the equation is called a **solution** or **root** of the equation. In other words, Left Hand Side of the equation (L.H.S.) = Right Hand Side of the equation (R.H.S.).

Example 1 : Verify that $x = 2$ is the solution of the linear equation $2x + 6 = 10$.

Solution : Put $x = 2$ in the L.H.S.

L.H.S. $= 2x + 6 = (2 \times 2) + 6 = 4 + 6 = 10$

R.H.S. $= 10$

$\Rightarrow$ L.H.S. = R.H.S.

$\therefore \quad x = 2$ is the solution of the equation $2x + 6 = 10$.

RULES FOR SOLVING A LINEAR EQUATION

An equation remains same, if :

1. The same quantity is added to both sides of the equation.
2. The same quantity is subtracted from both sides of the equation.
3. Both sides are multiplied by the same non-zero number.
4. Both sides are divided by the same non-zero number.
5. By Transposition Method : Any term in an equation can be taken to the other side of '=' sign by simply changing its sign from (i) + to – (ii) – to + (iii) × to ÷ (iv) ÷ to ×.
   This process of changing sign is called **transposition**.
   Any of the above rules or a combination of the above rules can be used to solve an equation.

SOLVING EQUATIONS HAVING VARIABLE TERMS ON ONE SIDE AND NUMBER (S) ON THE OTHER SIDE

Example 2 : Solve the equation $2x - 6 = 0$ and check the solution.

Solution : $2x - 6 = 0$

$\Rightarrow 2x - 6 + 6 = 0 + 6$ [Adding 6 to both sides]

$\Rightarrow 2x = 6$

$\Rightarrow \frac{2x}{2} = \frac{6}{2}$ [Dividing both sides by 2]

$\Rightarrow x = 3$

Check : L.H.S. $= 2x - 6 = (2 \times 3) - 6 = 6 - 6 = 0$

R.H.S. $= 0$

$\Rightarrow$ L.H.S. = R.H.S.

So, $x = 3$ is the solution of the given equation.

Example 3 : Solve the equation $\frac{2x-7}{4} - \left(\frac{1-x}{6}\right) = 17$ and check the solution.

Solution : $\frac{2x-7}{4} - \left(\frac{1-x}{6}\right) = 17$

$\Rightarrow \frac{3(2x-7) - 2(1-x)}{12} = 17$

$\Rightarrow \frac{3(2x-7) - 2(1-x)}{12} \times 12 = 17 \times 12$ [Multiplying both sides by 12]

$\Rightarrow 3(2x-7) - 2(1-x) = 17 \times 12$

$\Rightarrow 6x - 21 - 2 + 2x = 204$

$\Rightarrow 8x - 23 = 204$

$\Rightarrow 8x - 23 + 23 = 204 + 23$ [Adding 23 to both sides]

$\Rightarrow 8x = 227$

$\Rightarrow \frac{8x}{8} = \frac{227}{8}$ [Dividing both sides by 8]

$$\Rightarrow \quad x = \frac{227}{8} = 28\frac{3}{8}$$

Check : L.H.S. $= \frac{2x-7}{4} - \left(\frac{1-x}{6}\right) = \frac{2 \times \frac{227}{8} - 7}{4} - \left(\frac{1 - \frac{227}{8}}{6}\right)$

$$= \frac{\frac{227}{4} - 7}{4} - \left(\frac{8 - 227}{8 \times 6}\right) = \frac{227 - 28}{4 \times 4} - \left(\frac{8 - 227}{8 \times 6}\right)$$

$$= \frac{199}{16} - \left(\frac{-219}{48}\right) = \frac{199}{16} + \frac{219}{48}$$

$$= \frac{597 + 219}{48} = \frac{816}{48} = 17$$

R.H.S. $= 17$

$\Rightarrow$ L.H.S. $=$ R.H.S.

So, $x = \frac{227}{8}$ or $28\frac{3}{8}$ is the solution of the given equation.

SOLVING EQUATIONS HAVING VARIABLE TERMS AND NUMBER(S) ON BOTH SIDES

In case where an equation consists of variable terms and numbers on both sides of equality, the method of solving involves transposition or shifting of variable terms on one side and numerals on the other.

Example 4 : Solve : $\frac{x-4}{7} - x = \frac{5-x}{3} + 1.$

Solution : $\frac{x-4}{7} - x = \frac{5-x}{3} + 1$

L.C.M. of L.H.S. is 7 and L.C.M. of R.H.S. is 3.

$$\therefore \quad \frac{x-4}{7} - \frac{x \times 7}{7} = \frac{5-x}{3} + \frac{1 \times 3}{3}$$

$$\Rightarrow \quad \frac{x-4}{7} - \frac{7x}{7} = \frac{5-x}{3} + \frac{3}{3}$$

$$\Rightarrow \quad \frac{x-4-7x}{7} = \frac{5-x+3}{3}$$

$$= \quad \frac{-6x-4}{7} = \frac{8-x}{3}$$

L.C.M. of 7 and 3 is 21. On multiplying both sides by 21, we get

$$21\left(\frac{-6x-4}{7}\right) = 21\left(\frac{8-x}{3}\right)$$

$\Rightarrow \quad 3(-6x-4) = 7(8-x)$

$\Rightarrow \quad -18x - 12 = 56 - 7x$

$\Rightarrow \quad -18x + 7x = 56 + 12$ [Transposing $-7x$ to L.H.S. and -12 to R.H.S.]

$\Rightarrow\ -11x = 68 \qquad \Rightarrow\ x = \frac{-68}{11}.$ [Transposing – 11 to R.H.S.]

Example 5 : Solve $p - \frac{1}{4}\left(p - \frac{2-p}{6}\right) = \frac{2p+8}{3} - 3$ and check the solution.

Solution : $p - \frac{1}{4}\left(p - \frac{2-p}{6}\right) = \frac{2p+8}{3} - 3$

L.C.M. of 4, 6 and 3 = 12. On multiplying both sides by 12, we get

$$12\left\{p - \frac{1}{4}\left(p - \frac{2-p}{6}\right)\right\} = 12\left(\frac{2p+8}{3} - 3\right)$$

$$\Rightarrow\ 12p - 3\left(p - \frac{2-p}{6}\right) = 4\,(2p+8) - 36$$

$$\Rightarrow\ 12p - 3p + 3\left(\frac{2-p}{6}\right) = 8p + 32 - 36$$

$$\Rightarrow\ 9p + \frac{2-p}{2} = 8p - 4$$

$$\Rightarrow\ 2\left(9p + \frac{2-p}{2}\right) = 2\,(8p - 4)$$ [Multiplying both sides by 2]

$$\Rightarrow\ 18p + 2 - p = 16p - 8$$

$$\Rightarrow\ 17p + 2 = 16p - 8$$

$$\Rightarrow\ 17p - 16p = -8 - 2$$ [Transposing 16p to L.H.S. and 2 to R.H.S.]

$$\Rightarrow\ p = -10$$

Check : L.H.S. $= p - \frac{1}{4}\left(p - \frac{2-p}{6}\right)$

$$= (-10) - \frac{1}{4}\left\{(-10) - \frac{2-(-10)}{6}\right\}$$

$$= (-10) - \frac{1}{4}\left\{(-10) - \frac{2+10}{6}\right\}$$

$$= (-10) - \frac{1}{4}\left(-10 - \frac{12}{6}\right) = (-10) - \frac{1}{4}(-10-2)$$

$$= (-10) - \frac{1}{4}(-12) = -10 + 3 = -7$$

R.H.S. $= \frac{2p+8}{3} - 3 = \frac{2\,(-10)+8}{3} - 3 = \frac{-20+8}{3} - 3$

$$= \frac{-12}{3} - 3 = -4 - 3 = -7$$

$\Rightarrow$ L.H.S. = R.H.S.

So, $x = -7$ is the solution of the given equation.

SOLVING LINEAR EQUATIONS BY CROSS-MULTIPLICATION METHOD

Let there be an equation in the form $\frac{ax+b}{cx+d} = \frac{p}{q}$.

First convert the equation to linear form by the method of cross-multiplication.

$$\frac{ax+b}{cx+d} = \frac{p}{q} \Rightarrow q\,(ax+b) = p\,(cx+d)$$

Now, find the solution by the method of transposition.

Example 6 : Solve : $\frac{3y+5}{2y+1} = \frac{1}{3}$.

Solution : $\frac{3y+5}{2y+1} = \frac{1}{3}$

$\Rightarrow 3\,(3y+5) = 1\,(2y+1)$ [Cross multiply]

$\Rightarrow 9y + 15 = 2y + 2$

$\Rightarrow 9y - 2y = 1 - 15$ [Transposing 15 to R.H.S. and $2y$ to L.H.S.]

$\Rightarrow 7y = -14$

$\Rightarrow \frac{7y}{7} = \frac{-14}{7}$ [Dividing both sides by 7]

$\Rightarrow y = -2$.

Exercise : 8 (A)

1. Solve the following linear equations and check the result :

(a) $\frac{x}{5} + 2 = \frac{7}{15}$ (b) $3x + 2 = 5$ (c) $\frac{2}{5}y = 8$

(d) $10.5\,x - 5.6 = 4.1$ (e) $\frac{x-1}{4} + \frac{x-2}{3} = 4\frac{1}{6}$ (f) $\frac{3}{4}(x-4) = 11$

2. Solve the following linear equations and check the result :

(a) $2x - x = 21 + 3\,(2x+1)$ (b) $\frac{2x}{3} - \frac{x}{5} = \frac{3x-10}{5}$

(c) $\frac{3}{8}(x-5) = 11 - 7x$ (d) $9.8x - 13.8 = 2.7 - \frac{1}{5}x$

(e) $12 - 9x = -3 - 5x$ (f) $8x - 5 + 3x = 9x - 11$

(g) $\frac{2-9y}{17-4y} = \frac{4}{5}$

(h) $\{(3x+5)+(x+2)\}^2 + \{(3x+5)-(x+2)\}^2 = 20x^2 - 78$

3. Solve the following linear equations :

(a) $\frac{a+b}{x-b} = \frac{a-b}{x+b}$ (b) $\frac{x+5}{2} + \frac{x-5}{3} = \frac{25}{6}$

(c) $\frac{x+2}{3} - \frac{x-3}{4} = 5 - \frac{x-1}{2}$ (d) $\frac{4x+3}{4} - \frac{2x-1}{3} = x + \frac{1}{3}$

(e) $\frac{5x-9}{6x} = \frac{3}{5}$ (f) $\left(\frac{x+1}{x+2}\right)^2 = \frac{x+2}{x+4}$

(g) $\frac{3x}{5} - \frac{2x}{7} = \frac{4}{35}$ (h) $\frac{4x+1}{3} + \frac{2x-1}{2} = 6 + \frac{3x-7}{5}$

TRICKY CHALLENGE!

- Find the root of the following linear equation and verify your answer :

$$\frac{2}{3}(4m-1)-\left(2m-\frac{1+m}{3}\right)=\frac{1}{3}m+\frac{4}{3}$$

APPLICATIONS OF LINEAR EQUATIONS TO WORD PROBLEMS

Many problems in our day-to-day life can be solved easily and quickly with the help of algebra. The practical use of algebra lies in finding the value of unknown quantities from the known ones.

The following steps will provide an easy guide to solve word problems :

Step I : Read and re-read the word problem till you understand what is provided and what is asked for.

Step II : Assign the unknown measures, the letters x, y, z, etc.

Step III : Convert the language of the word problem into simple mathematical statements.

Step IV : Form equations using the conditions given in the problem.

Step V : Solve the equations to find the values of the unknown measures.

Example 7 : The sum of two numbers is 105. If one exceeds the other by 17, find the numbers.

Solution : Let one number be x, then the other number is $x + 17$.

Given, sum of the two numbers = 105

So, $x + (x + 17) = 105$

$\Rightarrow 2x + 17 = 105$

$\Rightarrow 2x = 105 - 17$ [Transposing 17 to R.H.S.]

$\Rightarrow 2x = 88$

$\Rightarrow x = 44$

Thus, the two numbers are 44 and $44 + 17 = 61$.

Example 8 : The father is 24 years older than his son. In 4 years, he will be thrice as old as his son. Find their present ages.

Solution : Let the age of the son be x years.

Therefore, father's age = $(x + 24)$ years.

In 4 years, the son's age will be $(x + 4)$ years and the father's age will be $\{(x + 24) + 4\}$ years, *i.e.*, $(x + 28)$ years.

According to the given condition,

$x + 28 = 3(x + 4)$

$\Rightarrow x + 28 = 3x + 12$

$\Rightarrow x - 3x = 12 - 28$ [Transposing $3x$ to L.H.S. and 28 to R.H.S.]

$\Rightarrow -2x = -16$

$\Rightarrow x = \frac{-16}{-2} = 8$

Thus, son's age = 8 years and father's age = $8 + 24 = 32$ years.

Example 9 : The denominator of a fraction is 6 more than its numerator. If 2 is added to both the numerator and the denominator, the fraction becomes $\frac{1}{2}$. Find the fraction.

Solution : Let the numerator of the fraction be x. Then the denominator of the fraction $= x + 6$.

So, the fraction $= \frac{x}{x+6}$

If 2 is added to both the numerator and the denominator,

New numerator $= x + 2$

New denominator $= (x + 6) + 2 = x + 8$

$\therefore$ New fraction $= \frac{x+2}{x+8}$

According to the given condition, $\frac{x+2}{x+8} = \frac{1}{2}$

$\Rightarrow 2(x + 2) = 1(x + 8)$ [Cross multiply]

$\Rightarrow 2x + 4 = x + 8$

$\Rightarrow 2x - x = 8 - 4$ [Transposing x to L.H.S. and 4 to R.H.S.]

$\Rightarrow x = 4$

$\therefore$ Numerator $= 4$, Denominator $= 4 + 6 = 10$

$\therefore$ The fraction is $\frac{4}{10}$.

Example 10 : The perimeter of a rectangular swimming pool is 154 m. Its length is 2 m more than twice its breadth. What are the length and the breadth of the pool?

Solution : Perimeter $= 154$ m

Let its breadth be x m.

$\Rightarrow$ Length will be $(2x + 2)$ m.

Perimeter $(P) = 2(x + 2x + 2)$

$\Rightarrow 154 = 2(3x + 2)$

$\Rightarrow \frac{154}{2} = 3x + 2$

$\Rightarrow 77 = 3x + 2$

$\Rightarrow 77 - 2 = 3x$ [Transposing 2 to L.H.S.]

$\Rightarrow 75 = 3x$

$\Rightarrow x = \frac{75}{3} = 25$ m

Thus, breadth $= 25$ m and length $= (25 \times 2) + 2 = 52$ m.

Exercise : 8 (B)

1. Write three consecutive integers whose sum is 63?

2. Two numbers are in the ratio 3 : 8. If sum of the two numbers is 165, find the numbers.
3. Three consecutive integers are such that when they are taken in increasing order and multiplied by 2, 3 and 4 respectively, they add up to 74. Find these numbers.
4. The sum of two numbers is 90 and the greater number exceeds twice the smaller by 15. Find the numbers.
5. The perimeter of a rectangle is 30 metres. If its length exceeds its breadth by 3 metres, then find its length.
6. The present ages of Akansha and Tripti are in the ratio 4 : 5. Eight years from now, their ages will be in the ratio 5 : 6. Find their present ages.
7. The denominator of a rational number is greater than its numerator by 7. If the numerator is increased by 17 and the denominator is decreased by 6, the new number becomes 2. Find the original number.
8. One of the two digits of a two-digit number is two times the other digit. If we interchange the digits of this two-digit number and add the resulting number to the original number, we get 99. What is the original number?
9. Five years ago, Mukul's father was five times Mukul's age. At present, the sum of Mukul and his father's age is 40 years. What are the present ages of Mukul and his father?
10. Divide 150 into three parts such that the second number is five-sixths of the first and third number is four-fifths of the second.

TRICKY CHALLENGE!

- There are some rose flowers in a flower pot and some bees are hovering around. If one bee lands on each rose flower, one bee is left. If two bees land on each rose flower, one flower is left. Find the number of bees and the number of rose flowers in the flower pot.

SUMMARY OF THE CHAPTER

- A linear equation is an equation involving expressions of degree one.
- A linear equation, having only one variable, is called a linear equation in one variable.
- The value of a variable, which satisfies the equation is called a solution or root of the equation.

Review Of The Chapter

(Task For Summative Assessment)

1. Solve the following equations and check the answer :

(a) $5x - (3x - 1) = x - 4$

(b) $\frac{7}{3}x - 2 = \frac{11}{2}x + 15$

(c) $2(x - 3) + \frac{1}{3}(2x - 5) = 3$

(d) $3(x - 1) = 8$

(e) $(x - 2)(x + 5) + 12 = (x + 3)(x - 4) - 2$

(f) $\frac{x}{3} + \frac{x}{4} + \frac{x}{5} = 1$

2. Solve the following equations by using cross–multiplication :

(a) $\frac{2}{3x} - \frac{3}{2x} = \frac{1}{12}$

(b) $\frac{7x - 1}{4} - \frac{1}{7}\left(2x - \frac{1 - x}{2}\right) = 4$

(c) $\dfrac{2x+3}{3x+4} = \dfrac{2x-3}{3x-2}$

(d) $4(x-1) + 2 = \dfrac{(x+5)}{2}$

(e) $\dfrac{4x+7}{9-3x} = \dfrac{1}{4}$

(f) $\dfrac{x}{3} + 1.5 - 0.12x = \dfrac{(0.5+x)}{2}$

3. If Tanu's age is two years less than twice her sister Meenu's age and the sum of twice Tanu's age and thrice Meenu's age is 66, then Tanu is 10 years old. Is it true or false?

4. The sum of two numbers is 40. If one of them is 10 more than the other, find the numbers.

5. Divide the share 64 between Riya and Tiya such that 3 times Riya's share is greater than 4 times Tiya's share by 10.

Multiple Choice Questions (MCQs)

(Task For Formative Assessment)

1. Which of the following is not a linear equation?
   (a) $2x + 7 = 10$ ☐ (b) $3x^2 = 9$ ☐
   (c) $a + b - 2 = 0$ ☐ (d) $\dfrac{4}{x} = 5$ ☐

2. The value of x for the equation $x + 9 = 10$ is :
   (a) 1 ☐ (b) 2 ☐
   (c) 19 ☐ (d) $\dfrac{10}{9}$ ☐

3. The solution of the equation $\dfrac{x-3a}{3} = 0$ is :
   (a) $x = 3a - 3$ ☐ (b) $x = 3a$ ☐
   (c) $x = 3a + 3$ ☐ (d) $x = -3a$ ☐

4. Which of the following is a linear equation?
   (a) $ax^2 + 4 = 0$ ☐ (b) $3x + 4 = 0$ ☐
   (c) $8x + 9$ ☐ (d) $x^3 = 8$ ☐

5. While solving an equation the same number or expression can be :
   (a) added or subtracted on both sides ☐
   (b) multiplied on both sides ☐
   (c) divided, if non-zero on both sides ☐
   (d) all of these ☐

6. If 5 less than a number is 72, then the number is :
   (a) 55 ☐ (b) 66 ☐
   (c) 77 ☐ (d) 88 ☐

7. A number when added to its half gives 72. The number is :
   (a) 48 ☐ (b) 84 ☐
   (c) 90 ☐ (d) none of these ☐

8. If $b \neq d$, then $\dfrac{ax+b}{cx+d} = \dfrac{b}{d}$, if :
   (a) only $a = b = 0$ ☐ (b) only $a = c = 0$ ☐
   (c) only $x = 0$ ☐ (d) all of these ☐

Maths Activity

Aim : To understand the concept of 'Magic Squares'.

Process : In magic squares, the sum of each row, column and diagonal is equal to the magical number for that square.

See an example :

Here, numbers 3 to 11 are written in the magical square such that the sum of each row, column and diagonal is a magical number 21.

10	3	8
5	7	9
6	11	4

Let's try another example :

	2	7
8		
	6	5

How can we find the missing numbers?

One way is by 'trial and error'. But there is another systematic way, which is by using 'linear equation'.

Let x be the missing number in row 1, column 1; y be the missing number in row 3, column 1; and m be the magical number.

x	2	7
8		
y	6	5

In row 1, $x + 2 + 7 = m \Rightarrow x + 9 = m$ .... (i)

In column 1, $x + 8 + y = m$ .... (ii)

Equating (i) and (ii), we get : $x + 8 + y = x + 9 \Rightarrow 8 + y = 9$

$\Rightarrow y = 9 - 8 = 1$

In row 3, $m = y + 6 + 5 = 1 + 6 + 5 = 12$

So, the magical number is 12.

In column 1, $x + 8 + y = m \Rightarrow x + 8 + 1 = 12 \Rightarrow x + 9 = 12 \Rightarrow x = 12 - 9 = 3$.

The other two missing numbers in row 2, column 2 and row 2, column 3 can be found in the same way. They are 4 and 0 respectively.

So, the complete magical square is :

3	2	7
8	4	0
1	6	5

Chapter 9

Percentage And Its Applications

Sequence of the Chapter

- Finding The Percentage Of A Number
- Increase Or Decrease In Percentage
- Profit And Loss
- Discount And Taxes

INTRODUCTION

In previous classes, we have learnt about percentage. We have learnt what does percentage mean. We have also learnt how to express a fraction, a ratio or a decimal as per cent and vice-versa. We have also learnt simple applications of percentage. Now in this chapter, we shall learn applications of percentage to advanced problems in profit and loss, discount, taxes, etc. Before proceeding further with the topic, let us first recall the basics we have learnt earlier.

★ 'Per cent 'means' per hundred'. Thus, 20% means 20 per hundred or 20 out of hundred.

★ Fractions and decimals can both be converted into percentages by multiplying the fraction or the decimal by 100. Thus the fraction $\frac{2}{5} = \frac{2}{5} \times 100\% = 40\%$.

Also, the decimal $0.30 = 0.30 \times 100\% = 30\%$.

★ A ratio can be converted into a percentage by first writing the ratio as a fraction and then multiplying it by 100. *For example,* $4 : 5 = \frac{4}{5} = \left(\frac{4}{5} \times 100\right)\% = 80\%$.

★ Percentages can be converted to decimals and fractions. *For example,* $30\% = \frac{30}{100} = 0.30$ (decimal form) $= \frac{3}{10}$ (fraction form).

★ Percentages can be expressed as ratios by first converting the percentages into fractions and then writing the fractions as ratios. *For example,* $75\% = \frac{75}{100} = \frac{3}{4} = 3 : 4$.

FINDING THE PERCENTAGE OF A NUMBER

To find the value of a given per cent of a given quantity, we multiply the given quantity by the fraction or decimal fraction of the given per cent, *i.e.,*

Value of a given per cent = Given quantity × Given per cent converted into fraction

Example 1 : What per cent of 20 is 80?

Solution : Let x be the required per cent.

Change x% into fraction, *i.e.*, $x\ \% = \frac{x}{100}$

So, $20 \times \frac{x}{100} = 80$

$$x = \frac{80 \times 100}{20} = 400$$

Therefore, $x = 400\%$.

Example 2 : If 60% people in a city like cricket, 30% like football and the remaining like other games, then what per cent of the people like other games? If the total number of people are 50 lakh, find the exact number who like each type of game.

Solution : 60% people like cricket and 30% like football.

$\Rightarrow$ 100% – (60 + 30)% people like other games or 10% people like other games.

Total number of people = 50 lakh

$\therefore$ Number of people who like cricket $= \frac{60 \times 50}{100}$ lakh = 30 lakh

Number of people who like football $= \frac{30 \times 50}{100}$ lakh = 15 lakh

Number who like other games $= \frac{10 \times 50}{100}$ lakh = 5 lakh

Example 3 : Jiya requires 40% to pass. If she gets 190 marks and falls short by 10 marks, what were the maximum marks?

Solution : Let the maximum marks be x.

Jiya was short by 10 marks in order to get 40%.

$\therefore$ 10 marks more than 190 = 190 + 10 = 200

$\therefore$ 40 % of x = 200

$\Rightarrow \frac{40}{100} \times x = 200$

$\Rightarrow 40 \times x = 200 \times 100$

$\Rightarrow x = \frac{200 \times 100}{40} = 500$

Hence, the maximum marks = 500.

INCREASE OR DECREASE IN PERCENTAGE

$$\text{Increase } \% = \left(\frac{\text{Increase}}{\text{Original value}} \times 100\right)\%$$

$$\text{Decrease } \% = \left(\frac{\text{Decrease}}{\text{Original value}} \times 100\right)\%$$

Example 4 : The enrolment of the students in a school increases from 900 to 936. Determine the per cent increase in the enrolment.

Solution : The increase in the number of students $= 936 - 900 = 36$

We have to determine what per cent of 900 is 36.

$\therefore$ Required percentage $= \dfrac{36}{900} \times 100 = 4\%$

Hence, the increase in enrolment of students is 4%.

Example 5 : Karan's salary was increased by 30%. A month later, his salary was increased by 30% . What is the net increase in Karan's salary?

Solution : Let Karan's salary be ₹ x.

The first increase $= 30\ \%$ of $x = \dfrac{30}{100} \times x = \dfrac{3x}{10}$

Thus, the total increased salary $= x + \dfrac{3x}{10} = \dfrac{13x}{10}$

A month later, his second increase $= 30\%$ of $\dfrac{13x}{10} = \dfrac{30}{100} \times \dfrac{13x}{10} = \dfrac{39x}{100}$

Now, this makes his total salary $= \dfrac{13x}{10} + \dfrac{39x}{100} = \dfrac{169x}{100}$

Thus, net increase $=$ final salary $-$ initial salary $= \dfrac{169x}{100} - x = \dfrac{69x}{100}$

Thus, the net per cent increase in salary $= \left(\dfrac{69x}{100} \times \dfrac{1}{x} \times 100\right)\% = 69\%.$

Exercise : 9 (A)

1. Convert the following into percentages :

(a) $\dfrac{1}{5}$ (b) $\dfrac{3}{4}$ (c) 5.5 (d) 0.9 (e) $2:3$

2. Convert the following percentages into fractions, ratios or decimals as indicated :

(a) 45% into decimal (b) 120% into decimal (c) $2\frac{1}{6}\%$ into fraction

(d) 135% into ratio (e) 150% into fraction (f) 0.25% into ratio

3. Find :

(a) 10% of 300

(b) 15% of ₹ 1500

4. What per cent is 10 of 50?

5. Tarun is batting in a cricket match. He has already scored 75% of the target runs. He needs to score 40 more runs to win. How many run has he already scored?

6. 35% of the population of a town are men and 40% are women. If the number of children is 20,000, find the number of women.

7. The salary of a clerk is increased by 30%. By what per cent must be increased salary be decreased in order to restore it to the former amount?

8. Mr. Rahul Kumar earns ₹ 14,950 per month. He is given a salary increment of ₹ 971.75. What is the percentage increase in Mr. Kumar's salary?
9. Kapil's salary is increased by 15% and then decreased by 10%. Find the percentage change in his salary.
10. In a town, water tax is increased by 20% and the consumption of water is decreased by 20%. What is the net effect on revenue from water tax?
11. Ritu requires 33% to pass. If she gets 155 marks and falls short by 10 marks, what are the maximum marks?
12. An alloy consists of 30% zinc, 45% nickel and the rest copper. Find the weight of copper in 800g of the alloy.

PROFIT AND LOSS

In our daily routine, we have to buy some articles from various shops. The shopkeepers purchase these articles either from wholesalers or directly from the manufacturers by paying a certain price. Generally, the shopkeeper sells his articles at a different price. These prices and difference in these prices are given special names such as **cost price**, **selling price**, **profit**, **loss**, etc. We have already studied about these terms in previous classes.

Let us revise them briefly and study some more facts about profit and loss.

Cost Price (C.P.)

The price for which an article is purchased is called the 'cost price' and abbreviated as C.P.

Selling Price (S.P.)

The price for which an article is purchased is called the 'selling price' and abbreviated as S.P.

Profit

If selling price is more than cost price (S.P.>C.P.), then the difference between the selling price and cost price is called the 'profit'.

∴ Profit = Selling Price – Cost Price

Loss

If selling price is less than cost price (S.P.<C.P.), then the difference between the selling price and cost price is called 'loss'.

∴ Loss = Cost Price – Selling Price

Overhead Expenses

We know that a retailer purchases goods from some manufacturer or wholesaler directly, and stores them in his shop or godown. For transporting the goods from the manufacturer or wholesaler, he spends some extra money. Sometimes he has to spend some money on repair and maintenance (especially for the second-hand items like a car, TV, etc.) of the goods to be sold. These additional expenses that he incurs are called 'overhead expenses'. The overhead expenses are added to the cost price to find actual profit (gain) or loss and the cost thus obtained is called the 'Net cost price' or 'Actual cost' or 'Total cost price'.

∴ Actual C.P. = Net C.P. = Price paid for goods + Overhead expenses

Generally, the profit or loss is expressed as profit% or loss%. It is to be kept in mind that the profit% or loss% is calculated on the cost price only.

Some Important Formulae To Find The Defined Terms Are As Follows :

Profit or Gain (S.P. > C.P.)

1. Profit = S.P. – C.P.
2. S.P. = Profit + C.P.
3. C.P. = S.P. – Profit
4. $\text{Profit\%} = \frac{\text{Profit}}{\text{C.P.}} \times 100$
5. $\text{Profit} = \frac{\text{C.P.} \times \text{Profit\%}}{100}$
6. $\text{S.P.} = \text{C.P.}\left(\frac{100 + \text{Profit\%}}{100}\right)$
7. $\text{C.P.} = \frac{100 \times \text{S.P.}}{(100 + \text{Profit \%})}$

Loss (S.P. < C.P.)

1. Loss = C.P. – S.P.
2. S.P. = C.P. – Loss
3. C.P. = Loss + S.P.
4. $\text{Loss\%} = \frac{\text{Loss}}{\text{C.P.}} \times 100$
5. $\text{Loss} = \frac{\text{C.P.} \times \text{Loss\%}}{100}$
6. $\text{S.P.} = \text{C.P.}\left(\frac{100 - \text{Loss\%}}{100}\right)$
7. $\text{C.P.} = \frac{100 \times \text{S.P.}}{(100 - \text{Loss\%})}$

Example 6 : Nishant bought a radio for ₹ 3,500. He sold it for ₹ 3,080. Find his loss per cent.

Solution : C.P. of the radio = ₹ 3,500

S.P. of the radio = ₹ 3,080

Since, C.P. > S.P., there is a loss.

Loss = C.P. – S.P. = ₹ (3,500 – 3,080) = ₹ 420

$$\text{Loss\%} = \left(\frac{\text{Loss}}{\text{C.P.}} \times 100\right)\% = \left(\frac{420}{3{,}500} \times 100\right)\% = 12\%$$

Hence, Nishant sold the radio at a loss of 12%.

Example 7 : If the selling price of 16 water bottles is equal to the cost price of 17 water bottles, find the gain per cent earned by the dealer.

Solution : Let the cost price of each water bottle be ₹ 1.

So, the cost price of 17 water bottles will be ₹ 17.

C.P. of 16 water bottles = ₹ 16

S.P. of 17 water bottles = ₹ 17

∵ S.P. > C.P.

∴ There is gain.

Gain = S.P. – C.P.

= ₹ 17 – ₹ 16 = ₹ 1

$$\text{Gain\%} = \frac{\text{Gain}}{\text{C.P.}} \times 100 = \frac{1}{16} \times 100 = \frac{25}{4} = 6\frac{1}{4}\%$$

Hence, the dealer gained $6\frac{1}{4}\%$.

Example 8 : The cost price of 10 tables is equal to the selling price of 8 tables. Find the loss or profit per cent.

Solution : Let the C.P. of each table = ₹100

Then, C.P. of 10 tables = ₹ 1000

Also, S.P. of 8 tables = ₹ 1000

So, S.P. of 1 table = ₹ $\frac{1000}{8}$ = ₹ 125

∴ Profit on 1 table = ₹ 125 – ₹ 100 = ₹ 25

Hence, Profit per cent = $\frac{25}{\cancel{100}} \times \cancel{100}\% = 25\%$.

Example 9 : Namit bought two horses at ₹ 20,000 each. He sold one horse at 15% gain. But he had to sell the second horse at a loss. If he had suffered a loss of ₹ 1,800 on the whole transaction, find the selling price of the second horse.

Solution : C.P. of 1 horse = ₹ 20,000

C.P. of 2 horses = ₹ 2 × 20,000 = ₹ 40,000   Given loss = ₹ 1,800

Total S.P. = C.P. – loss = ₹ 40,000 – ₹ 1,800 = ₹ 38,200

S.P. of 1st horse sold at 15% profit = $\left(\frac{100 + \text{Profit\%}}{100}\right) \times \text{C.P.}$

$= \left(\frac{100 + 15}{100}\right) \times 20{,}000 =$ ₹ 23,000

∴ S.P. of 2nd horse sold at loss = ₹ 38,200 – ₹ 23,000 = ₹ 15,200

Thus, the selling price of 2nd horse is ₹ 15,200.

Example 10 : If Rachna sells her teddy bear for ₹ 800, she would loss 20%. At what price must she sell her doll to gain 25%?

Solution : Let the C.P. of the doll be ₹ x. If S.P. of the doll is ₹ 800 , then loss is 20%.

$\text{Loss\%} = \left(\frac{\text{Loss}}{\text{C.P.}} \times 100\right)\% = \left(\frac{\text{C.P.} - \text{S.P.}}{\text{C.P.}} \times 100\right)\%$

$\Rightarrow \quad 20\% = \left(\frac{x - 800}{x} \times 100\right)\%$

$\Rightarrow \quad \frac{20}{100}x = x - 800$

$\Rightarrow \quad x - \frac{x}{5} = 800$

$\Rightarrow \quad \frac{4x}{5} = 800$

$\Rightarrow \quad x = 800 \times \frac{5}{4} =$ ₹ 1,000

Required gain % = 25% $= \left(\frac{\text{Gain}}{\text{C.P.}} \times 100\right)\%$

$\Rightarrow \quad \text{Gain} = \left(\frac{25}{100} \times \text{C.P.}\right) = \left(\frac{25}{100} \times 1{,}000\right) =$ ₹ 250

Thus, S.P. = C.P. + Gain = ₹(1,000 + 250) = ₹ 1,250

Hence, the required selling price of the doll is ₹ 1,250.

Exercise : 9 (B)

1. Tanisha bought a computer for ₹ 25,650. Due to some defect in it, she had to pay ₹ 1,350 for repair. Then she sold it for ₹ 28,700. Find her gain or loss per cent.
2. If the cost price of 10 chairs is equal to the selling price of 16 chairs, find the profit or loss per cent.
3. A merchant bought 10 kg of rice at ₹ 70 per kg. He sold 5 kg at ₹ 100 per kg and the remaining at ₹ 80 per kg. Find his profit and the profit per cent.
4. Nitish purchased a T.V. set for ₹ 2,700 and spent ₹ 300 on its repairing. He then sold it to Ankur at a gain of 25%. Ankur sold it to Pramod at a loss of 10%. Find the C.P. for :
   (a) Ankur (b) Pramod
5. Tarun sells two radio sets for ₹ 2,288 each. On one he gains 10% and on the other he loses 10%. Find his gain or loss per cent.
6. A person suffers a loss of 12.5% by selling an article for ₹ 17.50. At what price must he sell the article to make a profit of 20%?
7. Ankush purchased 120 mangoes at the rate of ₹ 2 per mango. He sold 60% of the mangoes at the rate of ₹ 2.50 per mango and the remaining mangoes at the rate of ₹ 2 per mango. Find his profit per cent.
8. If a manufacturer gains 10%, the wholesaler 15%, and the retailer 25%, then what is the cost production of an electric toaster whose retail price is ₹ 1,265?

DISCOUNT AND TAXES

Discount

We read advertisements in our day-to-day life in newspapers, magazines, banners, posters given by various companies and shopkeepers declaring discounts such as :

"Off Season Discount",
"Grand Puja Discount",
"Goods at Throw away prices",
"Now get 1200 g Desi Ghee for the cost of just 1 kg",
"Get a Steel Glass free with every 500g pack of coffee", etc.

When discount is given, a certain price is attached to the article which the shopkeeper professes to be the cost of the article for the customer. This price is called the **marked price** or **list price**. Then, the shopkeeper offers **discount** on this marked price. Customer pays the difference between the marked price and the discount. It is called **net price** or **discount price**.

Some useful formulae regarding discount, marked price, selling price, etc. are as follows :

1. Net Selling Price = Marked Price – Discount
2. Discount = Marked Price – Net Selling Price
3. Marked Price = Net Selling Price + Discount
4. Discount% = $\left(\frac{\text{Discount}}{\text{Marked Price}}\right) \times 100\%$
5. S.P. = $\text{M.P.} - \frac{\text{Discount\%} \times \text{M.P.}}{100}$

or, $S.P. = M.P.\left(1 - \frac{\text{Discount\%}}{100}\right)$ or, $S.P. = M.P.\left(\frac{100 - \text{Discount\%}}{100}\right)$

6. $M.P. = \frac{100 \times S.P.}{(100 - \text{Discount\%})}$

Taxes

When we purchase any article from the market, we have to pay a certain extra amount in addition to the marked price of the article. This extra amount is called the **sales tax**. These taxes are charged by the government to provide various facilities such as roads, transport, electricity, water, etc. to the general public.

Government has already fixed VAT (Value Added Tax) rates for various categories of items like clothes, medicines, footwears, etc., which we as customers (consumers) pay to the shopkeepers and the shopkeepers pay the same to the government on our behalf.

Just like the overhead expenses, taxes are also added to the cost price while dealing with problems concerning profit, loss, *etc.* Sales tax is always calculated on the net marked price of the article.

Example 11 : During a sale, a shopkeeper offered a discount of 10% on the marked prices of all the items. What would a customer have to pay for a pair of jeans marked at ₹ 1,450 and two shirts marked at ₹ 850 each?

Solution : Marked price of a pair of jeans = ₹ 1,450

Discount offered = 10% = ₹ $\frac{10 \times 1{,}450}{100}$ = ₹ 145

Price after discount = ₹ 1,450 – ₹ 145 = ₹ 1,305

Marked price of 1 shirt = ₹ 850

Marked price of 2 shirts = ₹ 1,700

Discount offered = 10% = $\frac{₹\ 10 \times 1{,}700}{100}$ = ₹ 170

Price after discount = ₹ (1,700 – 170) = ₹ 1,530

So, amount a customer has to pay for a pair of jeans and two shirts

= ₹ 1,305 + ₹ 1,530 = ₹ 2,835.

Example 12 : A dealer marks a washing machine 20% above the cost price of ₹ 8,000 and allows a discount of 10%. Find the selling price and the profit per cent.

Solution : C.P. of the refrigerator = ₹ 8,000

20% of C.P. = ₹ $\left(\frac{20}{100} \times 8{,}000\right)$ = ₹ 1,600

M.P. = C.P. + ₹ 1,600 = ₹ (8,000 + 1,600) = ₹ 9,600

Discount% = 10%

S.P. = ₹ $\left(\frac{100 - \text{Discount\%}}{100}\right) \times M.P. = ₹ \left(\frac{100 - 10}{100}\right) \times 9{,}600$

$= ₹ \left(\frac{90}{100} \times 9{,}600\right) = ₹\ 8{,}640$

Since S.P. > C.P., hence there is a profit.

Profit = S.P.– C.P. = ₹ (8,640 – 8,000) = ₹ 640

$$\text{Profit }\% = \frac{\text{Profit}}{\text{C.P.}} \times 100 = \frac{640}{8{,}000} \times 100 = 8\%$$

∴ Selling price of the refrigerator = ₹ 8,640 and profit % = 8%.

Example 13 : A shopkeeper offers 10% off-season discount to the customer and still makes profit of 26%. What is the cost price for the shopkeeper of a pair of shoes marked at ₹ 1,120?

Solution : Marked price of a pair of shoes = ₹ 1,120

Rate of discount = 10%

∴ Discount allowed = 10% of M.P.

$$= ₹\left(\frac{10}{100} \times 1{,}120\right) = ₹\ 112$$

Thus, S.P. of the pair of shoes = ₹ 1,120 – ₹ 112 = ₹ 1,008

Now, profit per cent of the shopkeeper = 26%

$$\therefore \text{C.P.} = \frac{100 \times \text{S.P.}}{100 + \text{Profit}\%} = ₹\left(\frac{100 \times 1{,}008}{100 + 26}\right)$$

$$= ₹\left(\frac{100 \times 1{,}008}{126}\right) = ₹\ 800$$

Thus, the cost price of the pair of shoes is ₹ 800.

Exercise : 9 (C)

1. Marked price of a pen is ₹ 20. It is sold at a discount of 15%. Find the discount allowed on the pen and its selling price.
2. (a) Find S.P., if M.P. = ₹ 700 and discount = 15%.

   (b) Find discount %, if M.P. = ₹ 1,200 and S.P. = ₹ 1,000.
3. A jacket was sold for ₹ 680 after allowing a discount of 15% on the marked price. Find the marked price of the jacket.
4. What price should Abdul mark on a pair of shoes which costs him ₹ 1,200 so as to gain 12% after allowing a discount of 16%?
5. Nitisha bought an air cooler for ₹ 5,500 including a tax of 10%. Find the price of the air cooler before VAT was added.
6. A shopkeeper offers two successive discounts of 10% and 5% on a saree whose marked price is ₹ 4,500. Find its selling price.
7. A shopkeeper allows 10% discount and still gains 10%. What is the cost price of a book which is marked at ₹ 220?
8. Tanu wants to buy a dress which is marked at a price of ₹ 4,050. VAT is applicable at 12.5%. However, Tanu does not want to pay more than ₹ 4,050. She requests the shopkeeper to offer her a discount so that she does not have to pay more than ₹ 4,050. Find the selling price that the shopkeeper will sell the dress at. Also find the discount % that the shopkeeper gives.

SUMMARY OF THE CHAPTER

- Value of a given per cent = Given quantity × Given per cent converted into fraction
- The price for which an article is purchased is called the cost price (C.P.).
- The price for which an article is purchased is called the selling price (S.P.).
- If selling price is more than cost price, then the difference between the selling price and cost price is called the profit.
- If selling price is less than cost price, then the difference between the selling price and cost price is called the loss.
- When discount is given, a certain price is attached to the article which the shopkeeper professes to be the cost of the article for the customer.

Review Of The Chapter

(Task For Summative Assessment)

1. Find :

(a) 60% of 700 g (b) $12\frac{1}{2}$% of ₹ 250

2. If 120% of x is 120, then find the value of x.

3. x is 5% of y, y is 24% of z. If $x = 480$, find the values of y and z.

4. A shopkeeper sold two tables for 1,485 each. On one he gains 10% and on the other he loses 10%. Find his gain or loss % in the whole transaction.

5. By selling a television set for ₹ 4,500, a dealer suffers a loss of 10%. Find the cost price of the television.

6. Find the M.P., if S.P. = ₹ 1,098 and Discount = 8.5%.

7. Rahul buys old books from his friend for ₹ 550. He spends ₹ 50 in binding them. He then sells them off to another friend for ₹ 670. Find the gain or loss per cent.

8. The marked price of a jeans and a shirt is ₹ 980. The VAT on jeans is 10% and that on the shirt is 5%. If the total VAT is ₹ 94, find the marked price of each of the items.

Multiple Choice Questions (MCQs)

(Task For Formative Assessment)

1. The decimal equivalent of $5\frac{1}{2}$% is :

(a) 0.55 ☐ (b) 0.055 ☐ (c) 0.0055 ☐ (d) 5.5 ☐

2. If 90% of x is 360, then x is :

(a) 320 ☐ (b) 324 ☐ (c) 400 ☐ (d) 450 ☐

3. An article marked at ₹ 1,200 is sold at 8% discount. The S.P. of the article is :

(a) ₹ 1,120 ☐ (b) ₹ 1,280 ☐ (c) ₹ 1,100 ☐ (d) ₹ 1,104 ☐

4. Sayali saves ₹ 25 by buying a skirt on sale. If she spends ₹ 250, the discount offered in the sale was about :

(a) 9% ☐ (b) 10% ☐ (c) 11% ☐ (d) 12% ☐

5. The cost price of 10 apples is the same as the selling price of 9 apples. The profit per cent is :

(a) 10% ☐ (b) $11\frac{1}{9}\%$ ☐

(c) $20\frac{2}{9}\%$ ☐ (d) 90% ☐

6. Ten per cent of twenty plus twenty per cent to ten equals :

(a) 1 per cent of 200 ☐ (b) 10 per cent of 20 ☐

(c) 2 per cent of 200 ☐ (d) 20 per cent of 10 ☐

7. Akash purchased a CD player for ₹ 1,760 including VAT. If the rate of VAT is 10%, the S.P. of the CD player is :

(a) ₹ 1,600 ☐ (b) ₹ 1,640 ☐

(c) ₹ 1,650 ☐ (d) ₹ 1,700 ☐

8. A chair with marked price ₹ 1,200 was sold to a customer for ₹ 1,000. The rate of discount allowed on the chair is :

(a) 11.2% ☐ (b) 12.25% ☐

(c) 13.33% ☐ (d) 16.66% ☐

9. If 45% of the population of a town are men and 30% are women, then the percentage of children is :

(a) 10% ☐ (b) 25% ☐

(c) 35% ☐ (d) 50% ☐

10. A shopkeeper buys x kg sugar at ₹ y per kg and sells it at ₹ z per kg. The formula for profit P is :

(a) $P = xz - xy$ ☐ (b) $P = 2xy - xy$ ☐

(c) $P = xy - xz$ ☐ (d) $P = yz - xy$ ☐