

Introduction To Graphs

Sequence of the Chapter

- Axis System (Cartesian System)
- ✓ Construction Of Line Graphs
- ✓ Linear Graphs



We have already learnt the tabulation method of arranging data and its graphical representation in a bar graph and pie chart. The graphical or pictorial method utilizes various other tools like histogram and line graph about which we will learn in this lesson.



Bar graphs are generally prepared for ungrouped data. To represent the distribution of a grouped data, we use histograms. A **histogram** is nothing but a pictorial representation of the grouped data in the form of rectangles of equal width, close to each other, whose bases represent the class and whose heights correspond to the frequencies.

Drawing A Histogram

To draw the histogram of grouped data, follow the given steps:

Step I: Find the lowest and the highest variable in the raw data.

Step II: Calculate the range.

Step III: Decide the number of classes you want to have and then the class-interval.

Step IV: Prepare the frequency distribution table by using tally marks.

Step V: Draw the two perpendicular axes.

Step VI: Mark class-intervals on the X-axis without leaving a gap (space) in between.

Step VII: Mark frequencies on the Y-axis by taking a suitable scale.

Step VIII: Draw rectangles on each class-interval according to the frequency.

Step IX: Write the title of the histogram and the names of the items represented by the horizontal and vertical axes.

Example 1: The raw data regarding the weights (in kg) of 30 students is given below: 60, 54, 62, 59, 41, 50, 37, 61, 47, 51, 63, 38, 68, 51, 60, 46, 68, 52, 38, 68, 59, 53, 60, 65, 59, 63, 69, 58, 58, 68

Prepare a frequency table using an approximate class-interval and draw the histogram.

Solution:

Writing the data in ascending order, we get

 $37,\,38,\,38,\,41,\,46,\,47,\,50,\,51,\,51,\,52,\,53,\,54,\,58,\,58,\,59,\,59,\,59,\,60,\,60,\,60,$

61, 62, 63, 63, 65, 68, 68, 68, 68, 69

Lowest value = 37, Highest value = 69

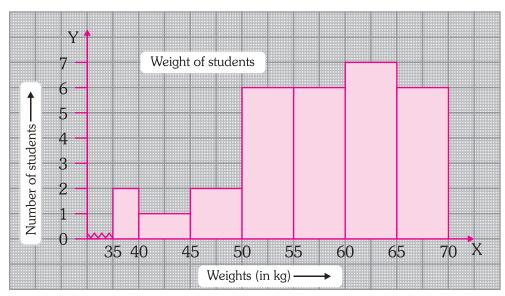
 \therefore Range = 69 - 37 = 32

Now, we take class-intervals like 35-40, 40-45, 45-50, 65-70.

Thus, the frequency distribution table is as follows:

Class-interval	Tally marks	Frequency
35 – 40		2
40 – 45		1
45 – 50		2
50 – 55	 	6
55 – 60	1111	6
60 – 65		7
65 – 70	1111	6
		Total = 30

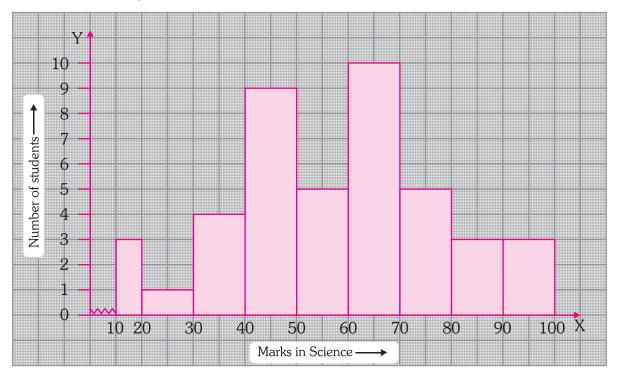
As classes are not beginning from 0, we use a kink $(\)$ to show this gap. On the basis of the table given above, the following histogram is drawn as follows:



Example 2: Study the following histogram and answer the questions given below:

- (a) What information is depicted in the above histogram?
- (b) What is the size of each class?
- (c) Write the number of students in the highest marks group.

- (d) In which group (s) the number of students is the same?
- (e) What is the number of students in the lowest marks group?
- (f) How many students have secured 60 or more marks?



- **Solution**: (a) The above histogram depicts the marks obtained by 43 students in Science.
 - (b) The size of each class is 10.
 - (c) The number of students in the highest marks group (90-100) is 3.
 - (d) The number of students is same in 50-60, 70-80 and 10-20, 80-90, 90-100.
 - (e) There are 3 students in the lowest marks group (10-20).
 - (f) Number of students who have secured 60 or more marks is 21 (10 + 5 + 3 + 3).



1. The weekly wages of 30 workers in a factory are :

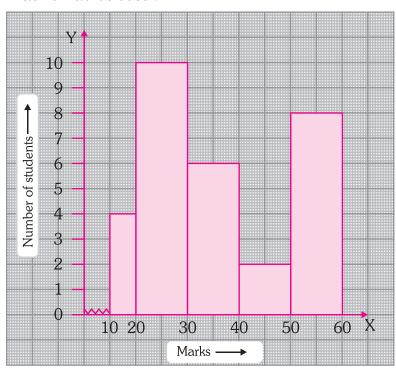
830, 835, 810, 836, 890, 835, 869, 845, 898, 890, 820, 868, 832, 833, 845, 812, 836, 855, 804, 840, 868, 885, 806, 890, 840, 835, 820, 869, 810, 835

Draw the grouped frequency distribution table and histogram. Take equal class-width and one of the class as 800-810.

2. Draw a histogram for the following table:

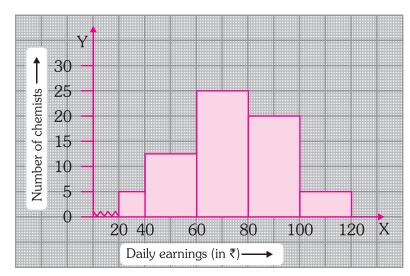
Class-interval	10 – 15	15 – 20	20 – 25	25 – 30	30 – 35
Frequency	30	98	80	58	29

- 3. The daily minimum temperatures of 30 days in a city are recorded as below. Organize the data such that one of the class-intervals is 38-40. Represent the data as a histogram. 34, 36, 38, 38, 36, 37, 35, 39, 37, 35, 36, 39, 36, 38, 42, 41, 41, 43, 38, 40, 41, 40, 35, 36, 38, 38, 37, 39, 42
- 4. The following histogram depicts the marks obtained by the students of class VIII in a Mathematics test:



Answer the following questions:

- (a) What is the class-size?
- (b) Which class-interval has the maximum number of students?
- (c) How many students scored less than 30 marks?
- (d) What is the class-interval in which the number of students is minimum?
- 5. The following histogram depicts the earnings of 70 chemists. Study it and answer the questions given ahead:



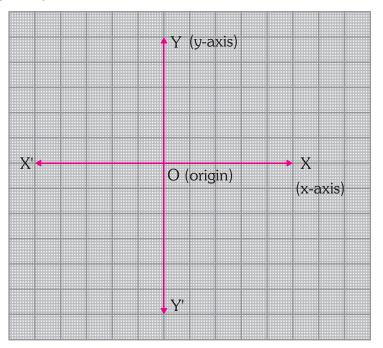
- (a) What is the class-size?
- (b) Which is the group with maximum earnings?
- (c) Which is the group with minimum earnings?
- (d) How many chemists earn less than ₹ 60 per day?

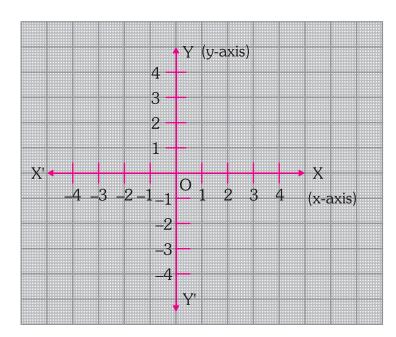
AXIS SYSTEM (CARTESIAN SYSTEM)

A point has 0 dimension and a plane figure has two dimensions. We require two reference lines (just like number lines) to describe the position of a point on a plane. These reference lines are called **axes**.

The horizontal line (from left to right) is called **x-axis**. The vertical line (from bottom to top) is called **y-axis**. The point where both the axes meet is called **origin**.

Together x-axis and y-axis are called **coordinate axes** and the plane containing the coordinate axes is called **cartesian plane** or **coordinate plane**.





- ★ On the x-axis, i.e., X'OX, OX denotes the positive part and OX' the negative part.
- \star On the y-axis, i.e., Y'OY, OY denotes the positive part and OY' the negative part.

Quadrants

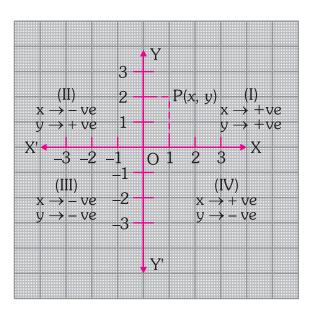
The coordinate axis divides the plane of the graph paper into four regions called **quadrants**.



Coordinates Of A Point

Let P be a point on the graph paper such that P is at a distance of x units from the y-axis and y units from the x-axis. Then the coordinates of P are (x, y). Here, x is called the x-coordinate or abscissa and y is called the y-coordinate or ordinate of P.

A point which lies on the x-axis has its y-coordinate zero and a point which lies on the y-axis has its x-coordinate zero.



Plotting Of Points

- Step I: On a graph paper, draw the x-axis and the y-axis and mark the origin.
- Step II: Choosing a suitable scale on x-axis and y-axis, mark the points on both the axes.
- Step III: Obtain the coordinates of the point which is to be plotted.
- Step IV: Keeping in mind the signs of the coordinates and the quadrant in which it lies, mark the point to be plotted. Repeat this step for all the points to be plotted.

Region	Quadrant	Sign of Coordinates
XOY	I	(+, +)
YOX'	II	(-, +)
X'OY'	III	(-, -)
Y'OX	IV	(+, -)

Keep In Mind!

In any given graph, there are small divisions between 0 and 1, 1 and 2, -1 and 0, -2 and -1, etc. on both x-axis and y-axis. So, the scale can be chosen in the given way.

Scale: X-axis: 10 small divisions = 1 unit

Y-axis:10 small divisions = 1 unit

Example 3: Determine the quadrant:

(a) A(6, -2)

(b) B(-3, -4)

(c) C(-3, 4)

(d) D(1, 2)

Solution:

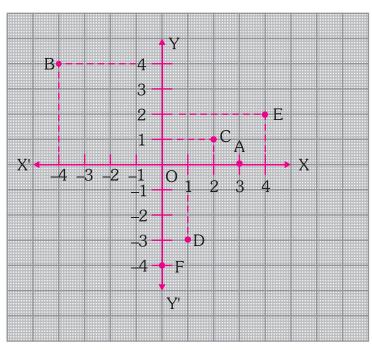
- (a) Since the x-coordinate of point A(6, -2) is positive and y-coordinate is negative, the point lies in the IV quadrant.
- (b) Both the coordinates of point B(-3, -4) are negative. So, it lies in the III quadrant.
- (c) The x-coordinate of C (– 3, 4) is negative and y-coordinate is positive. So, it lies in the II quadrant.
- (d) Since both the coordinates of point D(1, 2) are positive, it lies in the I quadrant.

Example 4: Write the coordinates of the following points shown on the graph paper:

- (a) *A*
- (b) B
- (c) C
- (d) D
- (e) E
- (f) F

Solution: (a) A(3,0)

- (b) B(-4,4)
- (c) C(2, 1)
- (d) D(1, -3)
- (e) E(4,2)
- (f) F(0, -4)



Example 5: Draw the following points on the same graph paper:

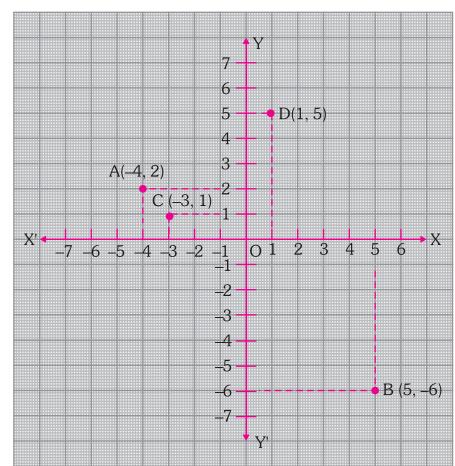
(a) A(-4, 2)

(b) B(5, -6)

(c) C(-3, 1)

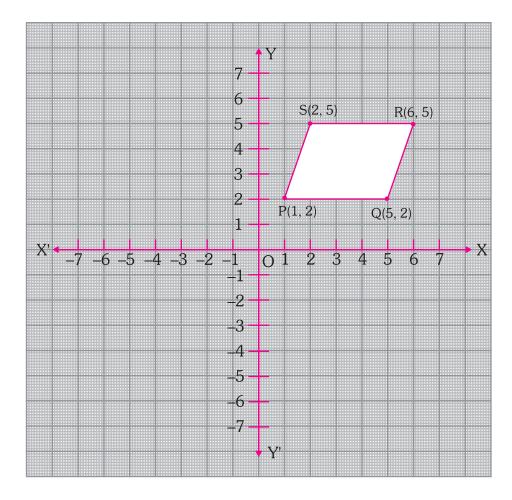
(d) D (1, 5)

Solution:



Example 6: Plot the following points on a graph paper: P(1, 2), Q(5, 2), R(6, 5) and S(2, 5). Join these points in order, i.e., P to Q, Q to R and so on. What figure do you get?

Solution:



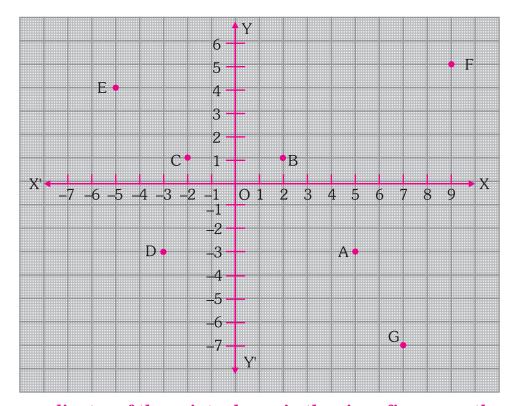
From the graph, the figure so obtained is a parallelogram, i.e., PQRS is a parallelogram.



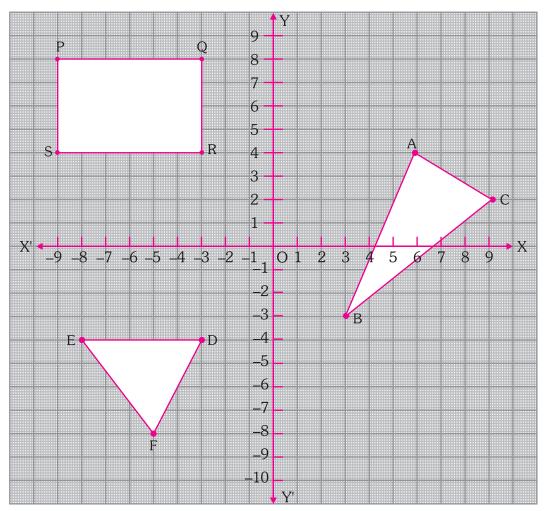
- 1. Plot the following points on graph and locate the quadrants in which these points lie:
 - (a) A (7, 6)
 - (c) C(2, 2)
 - (e) E(-5, -5)
 - (g) G(1, -4)

- (b) B(-3,3)
- (d) D(5, -5)
- (f) F(6, -9)
- (h) H(-6, -8)
- 2. Plot the following points on a graph:
 - (a) P(-6,0)
 - (c) R(3,0)

- (b) Q(0, -6)
- (d) S(-3,0)
- 3. Find the coordinates of the points marked on the graph given ahead:



4. Write the coordinates of the points shown in the given figures on the graph:





A **line graph** is a graph which is used to represent data that changes continuously with time.

While constructing line graphs, the values of one variable correspond to different values of the other variable. The variable whose value does not depend on the values of the other variable is called an independent variable and the variable whose values depend on the values taken by the other variable is known as a dependent variable.

To draw the graph of a given data, follow the given steps:

Step I: Identify independent and dependent variables from the given information.

Step II: Label the independent variables along OX and dependent variables along OY.

Step III: Choose an appropriate scale for plotting the points.

Step IV: Plot each ordered pair and join the plotted points to get the desired graph.

[Note: You can choose different scales along OX and OY.]

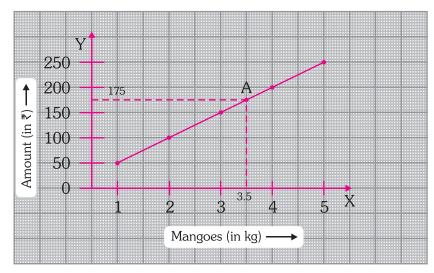


We have seen that a line graph is drawn by joining line segments. If the line graph is such that the lines drawn (in the line graph) form one whole unbroken straight line, then it is called a **linear graph**.

The following table gives the quantity of mangoes and its cost.

Quantity of mangoes (in kg)	1	2	3	4	5
Cost (in ₹)	50	100	150	200	250

If we represent this data as a line graph, we get the following graph:



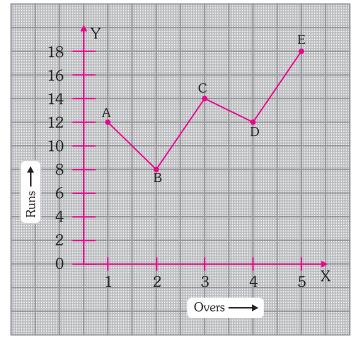
The above figure shows a straight line, *i.e.*, this is linear graph. The graph can be used to find the cost of any amount of mangoes. For example, if we want to find the cost of $3.5 \, \text{kg}$ of mango, we first find the point $3.5 \, \text{on}$ the x-axis. We then locate the point at which the vertical line through $3.5 \, \text{meets}$ the line on the graph, say A. We now find the point on the y-axis where the horizontal line through the point A meets the y-axis, say B. This point B on the y-axis gives the cost of B0 mangoes, which is B175.

Example 7: Draw a line graph to represent the given data.

Overs	1	2	3	4	5
Runs	12	8	14	12	18

Solution: x-axis: 20 small divisions = 1 unit (i. e., 1 over)

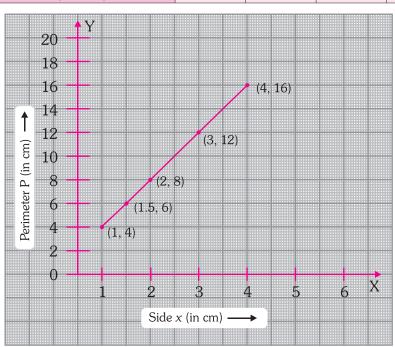
y-axis : 10 small divisions = 2 units (i. e., 2 runs)



Example 8: Draw a graph to represent the relationship between the perimeter and the side x of a square given as "Perimeter (P) = $4 \times$ side, i.e., P = 4x" using the following table:

Side of square, x (in cm)	1	1.5	2	3	4
Perimeter P (in cm)	4	6	8	12	16

Solution:



Math-8 (260)

To sketch the graph, take the side of square x (in cm) on the x-axis and the perimeter P (in cm) on the y-axis as per the following scale :

Scale: x-axis: 20 small divisions = 1 unit (i.e., 1 cm) y-axis: 10 small divisions = 2 units (i.e., 2 cm)

Example 9: Draw the graph between side and area of a square.

Solution: Let x =side of a square, y =area of the square

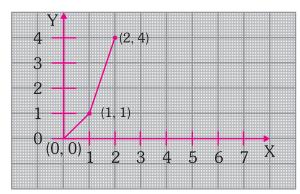
Clearly, we get an equation $y = x^2$, as area of square = (side)². Now, we calculate different values of y for different values of x.

When
$$x = 0$$
, $y = (0)^2 = 0$.
When $x = 1$, $y = (1)^2 = 1$.
When $x = 2$, $y = (2)^2 = 4$.
We tabulate the results as follows:

X	0	1	2
y	0	1	4
(x, y)	(0, 0)	(1, 1)	(2, 4)

Now, we plot the points on a graph paper as follows:

Scale: Each big square = 1 unit on both axes



Example 10: Kareena deposited ₹ 12,000 in a bank at the rate of 10% per annum. Draw a linear graph showing the relationship between the time and simple interest. Also, find the simple interest for 4 years.

Solution: Principal,
$$P = 7.2,000$$

Rate,
$$R = 10\%$$

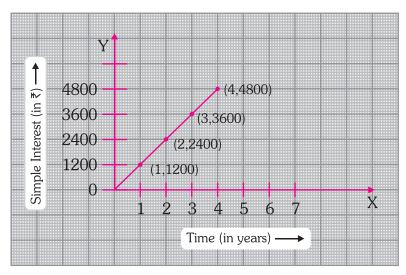
 $T = ?$
 $I = \frac{P \times R \times T}{100}$
 $\Rightarrow I = \frac{12,000 \times 10 \times T}{100}$
 $\Rightarrow I = 1200 T$

Here, variable I is dependent upon T.

For different values of T, the values of I are given in the table given ahead:

T (in yrs)	1	2	3	4
I (in ₹)	1200	2400	3600	4800

Based on the above table, plot a graph by taking the coordinates of the points as (1, 1200), (2, 2400), (3, 5600), and (4, 4800).



So, after 4 years Kareena will get ₹ 4,800 as simple interest.



1. Look at the following table which gives the quantity of petrol and its cost.

Number of litres of petrol	10	15	20	25
Cost of petrol (in ₹)	500	750	1000	1250

Plot a graph to show this data.

2. Draw a graph for the following data:

(a)	X	6	3	2	4	4
	у	1	2	3	4	5

(b)	x	1	2	3	4	5
	у	1	2	3	4	5

- **3.** Draw a graph for area of a circle with respect to its radius. [Take r = 7, 14, 21, 28]
- **4.** Draw a graph between simple interest versus time on amount ₹ 2,000 at the rate of interest 4% per annum.
- **5.** The table given below gives the temperature in °C and its corresponding value in °F.

Temperature (°C)	20	40	60	80	100
Temperature (°F)	60	100	140	180	210

Draw a graph by taking temperature in °C on x-axis and °F on y-axis.

SUMMARY OF THE CHAPTER

- A histogram is a pictorial representation of the grouped data in the form of rectangles of equal width, close to each other, whose bases represent the class and whose heights correspond to the frequencies.
- The coordinate axis divides the plane of the graph paper into four regions called quadrants.
- A line graph is a graph which is used to represent data that changes continuously with time.
- If the line graph is such that the lines drawn (in the line graph) form one whole unbroken straight line, then it is called a linear graph.



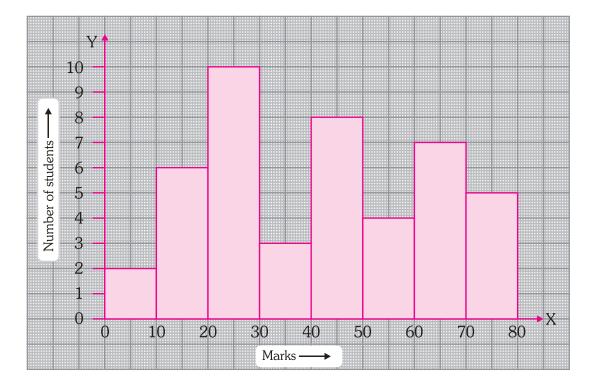
1. Draw a histogram for the following frequency table and answer the questions that follow:

Weekly wages	800–	810–	820 <u>–</u>	830–	840 <u>–</u>	850 <u>–</u>	860–	870–	880 <u>–</u>	890 <u>–</u>
	810	820	830	840	850	860	870	880	890	900
Number of workers	3	2	1	9	5	1	3	1	1	4

- (a) Which group has the maximum number of workers?
- (b) How many workers earn ₹850 and more?
- (c) How many workers earn less than ₹850?

2. The following histogram depicts the marks scored by 45 students of a class.

Look at the histogram and answer the questions given ahead:

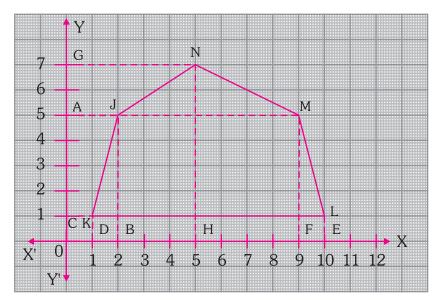


- (a) How many students scored less than 10 marks?
- (b) What is the class-size?
- (c) If passing marks are 40, what is the number of failures?
- 3. The sales (in ₹) of a shopkeeper on six days of the week are given below:

Days	1	2	3	4	5	6
Sales (in ₹)	4000	4500	3000	3500	4800	5000

Draw a line graph representing the above data.

- **4.** Draw a graph between side and volume of a cube.
- 5. Find the quadrants in which the following points lie:
 - (a) (1, 6)
- (b) (3, -6)
- (c) (-4, 4)
- (d) (-7, -2)
- 6. Write the coordinates of the vertices in the following figure:





Multiple Choice Questions (MCQs)

(Task For Formative Assessment)

(a) 1	1.	The number of coordinate axes	s is :					
(a) origin (b) abscissa (c) ordinate (d) quadrant (3. x-coordinate of a point is also called : (a) ordinate (b) origin (c) zero (d) abscissa (d) abscissa (d) none of these (e) (a) 0 (b) 5 (c) 3 (d) none of these (e) (c) 0 (d) none of these (e) (d) none of these (e) (d) none of these (e) (e) 0 (figure (d) none of these (e) (figure (d) none of these (e) (e) (figure (d) none of these		(a) 1 (b) 2	2	(c)	3	(d)	4	
3. x -coordinate of a point is also called : (a) ordinate	2.	The point where both axes mee	et is called :					
(a) ordinate		(a) origin (b) a	abscissa	(c)	ordinate	(d)	quadrant	
4. The ordinate of the point $(5,3)$ is: (a) 0	3.	x-coordinate of a point is also	called :					
(a) 0		(a) ordinate (b) c	origin	(c)	zero	(d)	abscissa	
5. The abscissa of the point $(-2, 6)$ is : (a) -2	4.	The ordinate of the point (5, 3)	is:					
(a) -2		(a) 0 (b) 5	5	(c)	3	(d)	none of these	
6. The point $(-5, -7)$ lies in :	5.	The abscissa of the point $(-2, 6)$	6) is :					
		(a) -2 (b) 6	6	(c)	0	(d)	none of these	
(a) 1st quadrant (b) 2nd quadrant (c) 3rd quadrant (d) 4th quadrant (6.	The point $(-5, -7)$ lies in :						
		(a) 1st quadrant (b) 2	2nd quadrant	(c)	3rd quadrant	(d)	4th quadrant	

The point $(0, 0)$ lies on the	e:				
(a) x-axis	(b) y-axis	(c) both axes	(d)	none of these	
The point $(5, 0)$ on the :					
(a) 1st quadrant	(b) 2nd quadran	t (c) x-axis	(d)	y-axis	
The equation of <i>y</i> -axis is	• •				
(a) $x = 0$	(b) $y = 0$		(d)	none of these	
The point which lies on the	ne y-axis has its :				
(a) both coordinates zero	0	(b) y-coordinate 2	zero		
(c) <i>x</i> -coordinate zero		(d) none of these			
	(a) x-axis The point $(5, 0)$ on the: (a) 1st quadrant The equation of y-axis is (a) $x = 0$ The point which lies on the	The point $(5, 0)$ on the : (a) 1st quadrant (b) 2nd quadrant The equation of y-axis is : (a) $x = 0$ (b) $y = 0$ The point which lies on the y-axis has its : (a) both coordinates zero	(a) x-axis	(a) x-axis	(a) x-axis



Aim: To understand the method of plotting points in a cartesian plane, and to show that ordered pairs are not reversible.

Materials required: Graph paper and geometry box.

Process:

- 1. Take a graph paper and draw two perpendicular lines as *XOX'* and *YOY'*. Mark the points at equal intervals.
- 2. All the given points are to be plotted in the order given. Accurate graphing of the ordered pairs produces the picture of a familiar object.

XI CONTRACTOR	0	
	Y'	

A (-2, 0)	B (- 1.5, 1)	C (0, 2)	D (4, 1.5)	E (7, 1)
F(9, 3)	G (8.5, 1.5)	H (8, 0)	I (8.5, – 1.5)	J(9, -3)
K (7, – 1)	L (4, – 1.5)	M(0, -2)	N (-1.5, -1)	O (-2, 0)

3. Let us reverse the ordered pair as follows:

A'(0,-2)	B' (1, – 1.5)	C' (2, 0)	D' (1.5, 4)	E' (1, 7)
F' (3, 9)	G' (1.5, 8.5)	H' (0, 8)	I' (-1.5, 8.5)	J' (-3, 9)
K' (-1, 7)	L' (-1.5, 4)	M' (-2, 0)	N' (-1, -1.5)	O' (0, – 2)

4. Take another graph paper and plot the reverse ordered pairs in the given order. After joining the points, a picture is formed. What is the difference between the two pictures?



Direct And Inverse Variations

Sequence of the Chapter

- ✓ Variations
- ▼ Direct Variation
- ✓ Inverse Variation

INTRODUCTION

We have already learnt about ratio and proportion in the previous class. 'Ratio' means comparing two quantities. For example, a basket contains 30 bananas and 40 apples.

Then, the ratio of number of bananas to the number of apples = 30 : 40 = 3 : 4 (simplest form) We generally compare the numbers in a ratio with a colon (:) and a 'proportion' is a way of stating that two ratios are equal.

For example, $\frac{30}{40} = \frac{3}{4}$ is a proportion and can be expressed as 30:40::3:4 and is read as '30

is to 40 is same as 3 is to 4'. Here, 30 and 4 are called extreme terms, while 40 and 3 are called mean terms.

In a proportion, we generally use double colon sign (::) which means 'the same as' and in a proportion the product of extremes is always equal to the product of means, i.e., $30 \times 4 = 40 \times 3 = 120$.

Hence,

Product of extremes = Product of means

Now, in this chapter, we shall learn about the variations in quantities, comparison of variation *i.e.*, ratio in which they vary and relation with unitary method. We'll also solve some problems related to time and work.

VARIATIONS

There are two types of variations:

- 1. Direct variation
- 2. Indirect or Inverse variation



Two quantities are said to be in direct variation if the increase (or decrease) in the measure of one results in the corresponding increase (or decrease) in the measure of the other.

Consider the following examples:

- ★ The more the distance a taxi covers, the higher is the taxi fare.
- ★ The more hours a man works, the higher are the earnings.
- ★ The lesser the speed of the vehicle you are in, the lesser the distance you will cover over a given period of time.

Constant Of Variation

If two quantities x and y vary in such a manner that $\frac{x}{y}$ is constant and positive, then x and y are

said to be in **direct variation**. In other words, x and y are in direct variation if $\frac{x}{y}$ is always constant. This constant is the constant of variation (say k).

$$\frac{x}{y} = k$$
 or $x = ky$

If two quantities x and y vary directly and x_1 and y_1 are their values at one point, then

$$\frac{x_1}{y_1} = k$$
, a constant (i)

If x_2 and y_2 are the values which x and y assume at second point, then

$$\frac{x_2}{y_2} = k$$
, a constant (ii)

Equating (i) and (ii), we get $\frac{x_1}{y_1} = \frac{x_2}{y_2}$

$$\Rightarrow x_1 y_2 = x_2 y_1$$

$$\Rightarrow \frac{x_1}{x_2} = \frac{y_1}{y_2}$$

$$\Rightarrow x_1:x_2::y_1:y_2$$

$$\Rightarrow x_1: x_2 = y_1: y_2$$

Example 1: Verify whether the following represent direct variation or not:

No. of pencils (x)	5	7	9	10	11	12
Cost (in ₹) (<i>y</i>)	25	35	45	50	55	60

Solution: Since
$$\frac{x}{y}$$

Since
$$\frac{x}{y} = \frac{\text{no. of pencils}}{\text{cost (in ₹)}} = \frac{5}{25} = \frac{1}{5}$$

$$\frac{x}{y} = \frac{7}{35} = \frac{1}{5}$$

$$\frac{x}{y} = \frac{9}{45} = \frac{1}{5}$$

$$\frac{x}{y} = \frac{10}{50} = \frac{1}{5}$$

$$\frac{x}{y} = \frac{11}{55} = \frac{1}{5}$$

$$\frac{x}{y} = \frac{12}{60} = \frac{1}{5}$$

$$\therefore \frac{x}{y}$$
 is constant and is equal to $\frac{1}{5}$.

Thus, no. of pencils and their cost are in direct variation.

Example 2: If one dozen bananas cost $\mathbf{\xi}$ 36, how many bananas can be bought for $\mathbf{\xi}$ 72.

Solution: Let us denote the required number of bananas be
$$x$$
.

For
$$\ref{36}$$
, number of bananas bought = 12.

Then, writing the like terms together, we have :

No. of bananas	Cost (in ₹)
12	36
X	72

Ratio of bananas =
$$\frac{12}{x}$$
 and ratio of costs in rupees = $\frac{36}{72}$

Since, more bananas cost more money, this is a case of direct variation.

Therefore,
$$\frac{12}{x} = \frac{36}{72}$$

or, $x = \frac{12 \times 72}{36} = 24$

Hence, the number of bananas bought for ₹ 72 is 24.

Example 3: If the weight of 6 sheets of paper is 45 grams, how many sheets of the same paper will weigh $1\frac{1}{2}$ kg?

Solution: Let *x* sheets of paper weigh
$$1\frac{1}{2} \text{ kg} = 1500 \text{ g}$$
.

Number of sheets	6	X
Weight (in grams)	45	1500

Since the number of sheets varies directly as the weight,

$$\therefore \frac{6}{45} = \frac{x}{1500}$$
or, $x = \frac{1500 \times 6}{45} = 200$

Hence, 200 sheets of paper will weigh $1\frac{1}{2}$ kg.

Example 4: Fill in the missing entries, if x and y vary directly:

х	6	9	4.5			24
у	10		7.5	12.5	20	

Solution: $\frac{6}{10}$

$$\frac{6}{10} = \frac{3}{5}$$
 and $\frac{4.5}{7.5} = \frac{3}{5}$

So, $\frac{3}{5}$ is the constant ratio.

Since *x* and *y* vary directly,

$$\therefore \frac{9}{y_1} = \frac{3}{5} \implies y_1 = \frac{9 \times 5}{3} = 15$$

$$\frac{x_1}{12.5} = \frac{3}{5} \implies x_1 = \frac{12.5 \times 3}{5} = 7.5$$

$$\frac{x_2}{20} = \frac{3}{5} \implies x_2 = \frac{20 \times 3}{5} = 12$$

$$\frac{24}{y_2} = \frac{3}{5} \implies y_2 = \frac{24 \times 5}{3} = 40$$

:. The missing entries are:

9	7.5	12	24
<u>15</u>	$\overline{12.5}$,	$\overline{20}$	40

Example 5: Karishma

Karishma has a road map with a scale of 1 cm representing 18 km. She drives on a road for 72 km. What would be her distance covered in the map?

Solution: Distance on map and actual distance are directly proportional.

Distance on map	Actual distance
1 cm	18 km
x cm	72 km

So,
$$1: x = 18:72$$

or,
$$1: x = 1: 4$$

or,
$$x = 4$$
 cm

Hence, Karishma covered 4 cm distance in the map.



1. Which of the following show direct variation?

- (a) Height of a person and the number of years
- (b) Income tax and the income
- (c) Number of people and quantity work
- (d) Number of rainy days and the amount of rainfall on those days
- (e) Height of a child and his weight as he grows

2. In which of the following is x in direct variation with y?

(a)	х	7	9	11	15
	у	28	36	44	60

(b)	х	2	1	6	7
	У	8	5	12	14

(c)	X	10	20	30	60	70	75
	У	2	4	6	12	14	15

(d)	x	1	3	7	10	13	16
	у	5	15	21	60	52	48

3. Complete the following tables assuming that x is in direct variation with y:

(a)	x	2	4			12
	У	3		12	15	

(b)	x	•••••	16	20		60
	У	2	•••••	5	8	15

- **4.** The variable x varies directly as y. When y is 10, x is $\frac{3}{2}$. Find y when x is 9.
- **5.** The cost of 9 oranges is ₹ 40.50. How many oranges can be bought for ₹ 292.50?
- **6.** If the weight of 5 boxes is 12.5 kg, find the weight of 9 such boxes.
- 7. If 5 men or 8 women earn ₹ 625 per day, how much would 8 men and 12 women earn per day?
- **8.** I can type 1920 words in 90 minutes. Find the time required to type 3008 words.
- 9. A car is travelling 50 km in one hour. How much distance would it travel in 12 minutes?
- **10.** The amount of extension in an elastic string varies directly with the weight hung to it. If a weight of 150 g produces an extension of 2.9 cm, what would produce an extension of 17.4 cm?

INVERSE VARIATION

Two quantities are said to be in inverse variation if the increase (or decrease) in the measure of one results in the corresponding decrease (or increase) in the measure of the other.

Consider the following examples :

- \star As the number of workers increases, the time taken to complete a given job decreases.
- * As the speed of a car increases, the time taken to cover the same distance decreases.
- ★ As the number of periods decreases in a school for a given day, the duration of each period increases (the number of school hours remaining same).

Constant Of Variation

If two quantities x and y vary in such a manner that xy is constant and positive, then x and y are said to be in **inverse** or **indirect variation**. In other words, x and y are in inverse variation if xy is always constant. This constant is the constant of variation (say k).

$$xy = k$$
 or $x = k\left(\frac{1}{y}\right)$

If two quantities x and y vary inversely and x_1 and y_1 are its values at one point, then

$$x_1y_1 = k$$
, a constant(i)

If x_2 and y_2 are the values which x and y assume at second point, then

$$x_2y_2 = k$$
, a constant (ii)

Equating (i) and (ii), we get $x_1y_1 = x_2y_2$

$$\Rightarrow \frac{x_1}{x_2} = \frac{y_2}{y_1}$$

$$\Rightarrow x_1:x_2::y_2:y_1$$

$$\Rightarrow x_1:x_2=y_2:y_1$$

Example 6: In the following table, find if x and y vary inversely:

х	4	6	12	20
у	30	20	10	6

Solution:

We know that if the product xy remains constant for all values of x and the corresponding values of y, then x and y will vary inversely. Let us find the product xy for all values.

$$4 \times 30 = 120$$
, $6 \times 20 = 120$, $12 \times 10 = 120$, $20 \times 6 = 120$

Since the product *xy* is constant, *x* and *y* vary inversely.

Example 7: If x and y vary inversely, then find the value of the missing numbers in the following table :

X	10	40	8	x_1
y	4	y_1	y_2	0.5

Solution:

$$xy = k$$

$$k = 10 \times 4 = 40$$

Now,
$$40 \times y_1 = 40$$
 \Rightarrow $y_1 = \frac{40}{40} = 1$
 $8 \times y_2 = 40$ \Rightarrow $y_2 = \frac{40}{8} = 5$
 $x_1 \times 0.5 = 40$ \Rightarrow $x_1 = \frac{40}{0.5} = 80$

Hence,
$$y_1 = 1$$
, $y_2 = 5$ and $x_1 = 80$.

Example 8:

In a boarding house of 80 boys, there is food provisions for 30 days. If 20 more boys join the boarding house, how long will the provisions last?

Solution:

Obviously, more the boys the sooner would the provisions exhaust. It is, therefore, the case of inverse variation. The numbers of boys in the two situations are 80 and 100 (80 + 20) respectively. If the provisions last for x

days when the number of boys increased from 80 to 100, we can have the following table :

Number of Boys	Number of Days
80	30
100	X

Here, the ratios between the like terms are $\frac{80}{100}$ and $\frac{30}{x}$.

Since the problem is of inverse variation, we will invert the ratio and then equate them.

i.e.,
$$\frac{x}{30} = \frac{80}{100} = \frac{4}{100}$$

or,
$$\frac{x}{30} = \frac{4}{5}$$
 or, $x = \frac{4 \times 30^{6}}{5} = \frac{4 \times 6}{1}$

or,
$$x = 24$$

Hence, the provisions will last for 24 days.



1. Which of the following are in inverse variation with each other?

- (a) Population of a country and area of land per person
- (b) Area of cultivated land and crop harvested
- (c) Time taken to cover a given distance with variation in speed of a vehicle
- (d) The number of chocolates (x) you can buy with $\stackrel{?}{\sim}$ 50 and the cost (y) of each chocolate
- (e) Number of people and time taken to complete a work

2. In which of the given tables, x and y are in inverse variation with each other?

(a)	x	5	3	2	30	6
	у	6	10	15	1	5

)	x	4	2	3	11	15
	у	11	12	22	14	6

(c)	x	10	2	3	16	5	6	
	y	6	15	12	10	12	11	

(d)	x	8	4	5	2	10	1
	у	10	20	16	40	8	80

3. Fill in the missing entries if x and y vary inversely:

(a)	х	42	4	14		28	21
	У	2	••••	•••••	12	•••••	4



- 4. If 40 goats can graze a field in 10 days, how many goats will graze the same field in 25 days?
- 5. 20 men build a wall in 10 days. In how many days will 25 men build the same wall?
- **6.** A car takes 2 hours to reach a destination by travelling at the speed of 60 km/h. How long will it take when the car travels at the speed of 80 km/h?

- **7.** A box of chocolates was distributed among 8 children and each child got 5 chocolates. If this box of chocolates is distributed among 10 children, how many chocolates will each child get?
- **8.** A school has 8 periods a day each of 50 minutes duration. How long would each period be, if the school has 10 periods a day, assuming the working hours of the school is same?
- 9. 8 taps having the same rate of flow, fill a tank in 27 minutes. If two taps go out of order, how long will the remaining taps take to fill the tank?
- **10.** A contractor undertook a contract to complete a part of a stadium in 9 months with a workforce of 560 men. Later on, it was required to complete the job in 7 months. How many extra men would he employ?



Problems On Time And Distance

We know that the speed of a moving body is the distance moved in unit time. It is usually represented either in m/s or km/h.

Relation among speed, time and distance:

The relation among speed, time and distance is given by

$$Distance = Speed \times Time$$

This relation connects three quantities, *i.e.*, distance, speed and time. If any two of them are given, it is easy to determine the third one.

By the above relation, we can also get:

Speed =
$$\frac{\text{Distance}}{\text{Time}}$$

$$Time = \frac{Distance}{Speed}$$



To solve problems related to time and distance, we can use principles of variation, such as:

- The greater the speed, the greater is distance travelled in particular time.
- When the speed remains constant, the distance travelled is directly proportional to the time.
- $\,\,\underline{\hspace{1.5pt}}$ The greater is the speed, the lesser the time taken to travel the distance.

Example 9: A man takes 11 hours to travel 36 kilometres. How long will he take to

travel 72 kilometres?

Solution: The man travels 36 km in 11 hours.

So, the man travels 1 km in $\frac{11}{36}$ hours.

The man travels 72 km in $\frac{11}{36^1} \times 72^2$ hours, i.e., in 22 hours.

Example 10: Two trains 300 m and 450 m long are going in the same direction at the speed of 20 km/h and 15 km/h on parallel tracks respectively. The shorter train is behind the longer train. How long would they take to pass each other, if the faster train is behind the slower train.

Solution: When trains are moving in the same direction, then the resultant speed is the difference of their speeds and the distance covered will be the sum of the lengths of the trains.

Speed = (20 - 15) km/h = 5 km/h

Distance covered = (300 + 450) m = 750 m = $\frac{750}{1000}$ km = $\frac{3}{4}$ km

.. Time to pass each other = $\frac{d}{s} = \frac{3}{4} \div 5 = \frac{3}{20}$ hr = $\frac{3}{20} \times 60$ min = 9 minutes.

Problems On Time And Work

To solve problems on 'time and work', we use the principles of direct and indirect variations, such as :

"More men do more work and less men do less work." (Direct variation)

"More men take less time to do a work and less men take more time to do the same work." (Indirect variation)

We use the unitary method by considering the following fundamental rules for solving problems regarding time and work :

1. Take a complete job or work to be 1.

2. Time to complete a work = $\frac{\text{Total work to be done}}{\text{Part of the work done in one day}}$

Example 11: Tiya, Ridhima and Preeti together complete a piece of work in 4 days. Tiya alone completes it in 12 days and Ridhima alone in 15 days. How many days will Preeti take to complete the work on her own?

Solution: Time taken by all three to complete the work = 4 days

Time taken by Tiya to complete the work =12 days

Time taken by Ridhima to complete the work = 15 days

Work done by all in one day = $\frac{1}{4}$

Work done by Tiya in one day = $\frac{1}{12}$

Work done by Ridhima in one day = $\frac{1}{15}$

Work done by Preeti in 1 day = Work done by all in 1 day - Work done by Tiya and Ridhima in 1 day

$$= \frac{1}{4} - \left(\frac{1}{12} + \frac{1}{15}\right) = \frac{1}{4} - \frac{1}{12} - \frac{1}{15}$$
$$= \frac{15 - 5 - 4}{60} = \frac{6}{60} = \frac{1}{10}$$

Since Preeti does $\frac{1}{10}$ th of the work in 1 day, she will complete the work alone in 10 days.

- **Example 12**: Karan and Mithil can together paint a wall in 25 minutes. Karan can alone paint the same wall in 50 minutes. They work together for 20 minutes, then Karan goes away. In how many minutes will Mithil paint the remaining part of the wall?
- **Solution**: Karan and Mithil can together paint a wall in 25 minutes.
 - \therefore Karan's and Mithil's 1 minute work = $\frac{1}{25}$

Karan can alone paint the wall in 50 minutes.

- \therefore Karan's 1 minute work = $\frac{1}{50}$
- .. Mithil's 1 minute work = (Karan's and Mithil's 1 minute work) – (Karan's 1 minute work) = $\frac{1}{25} - \frac{1}{50} = \frac{2-1}{50} = \frac{1}{50}$

The amount of work that can be done by Karan and Mithil in

$$20 \text{ minutes } = 20 \times \frac{1}{25} = \frac{4}{5}$$

Remaining work = $1 - \frac{4}{5} = \frac{1}{5}$

Mithil can complete $\frac{1}{50}$ of the work in 1 minute.

... Mithil can complete 1 work in $1 \div \frac{1}{50}$ minute = $1 \times \frac{50}{1}$ minutes = 50 minutes

Hence, $\frac{1}{5}$ of remaining work will be completed by Mithil in

$$50 \times \frac{1}{5} = 10$$
 minutes.

- \therefore Mithil will paint the remaining part of the wall in 10 minutes.
- **Example 13**: A can build a room in 50 days. A and B together can build a room in 25 days. They worked together for 15 days and then B goes away. In how many days will A finish the remaining work?
- **Solution**: Time taken by *A* to finish the work is 50 days.

A's 1 day's work = $\frac{1}{50}$ th of work

Together (A + B) can finish the work in 25 days.

So, (A + B)'s 1 day's work = $\frac{1}{25}$ th part of work

A and B work together for 15 days.

So,
$$(A + B)$$
's 15 day's work = $15 \times \frac{1}{25} = \frac{3}{5}$

Remaining work =
$$1 - \frac{3}{5} = \frac{2}{5}$$

Now $\frac{1}{50}$ th of work is done by A in 1 day.

$$\therefore \frac{2}{5}$$
 of work will be done by A in $\left(\frac{2}{5} \times 50\right)$ days.

Hence, the remaining work is done by *A* in 20 days.



- 1. A train of length 320 metres is running at a speed of 72 km/h. How much time will it take to cross a pole?
- 2. Two trains 350 m and 250 m long are going at 42 km/h and 30 km/h respectively in opposite direction. How long would it take them to pass each other?
- **3.** Tarun completes a piece of work in 2 days and Rahul completes the same work in 4 days. If they both work together, in how many days will they complete the work?
- **4.** Mohit, Honey and Ravi can reap a field in $\frac{63}{4}$ days; Honey, Ravi and Kapil in 14 days; Ravi, Kapil and Praveen in 18 days; Kapil, Praveen and Naman in 21 days. In how much time can all of them together reap the field?
- **5.** *A* and *B* together can do a piece of work in 10 days. *A* alone can finish the work in 30 days. In how many days will *B* alone finish the work?
- 6. Rohan can do a piece of work in 25 days and his friend Akram can do same work in 20 days. They started the work together, after 5 days Akram leaves the work. In how many days will Rohan complete the remaining work?
- **7.** *A*, *B* and *C* can do a piece of a work in 3 days, 4 days and 6 days respectively. How long will they take to complete it, if they work together?
- **8.** A and B can do a piece of work in 18 days, B and C can do it in 24 days, while C and A can finish it in 36 days. In how many days can A, B, C finish it, if they all work together?

SUMMARY OF THE CHAPTER

- In variation, if we change the value of one quantity, then there is a corresponding change in the value of the other quantity also.
- If two quantities x and y vary in such a manner that $\frac{x}{y}$ is constant and positive, then x and y are said to be in direct variation.
- If two quantities x and y vary in such a manner that xy is constant and positive, then x and y are said to be in inverse or indirect variation.

1. If x varies directly as y, then fill in the blanks:

(a)	X	5	5
	υ	15	

х	9	
у	36	72

2. If x varies inversely as y, then fill in the blanks:

(a)	х	•••••	30
	у	3	2

(b)			
(b)			
	X	1	

		0.4	
	y	<i> 2</i> 4	0

- **3.** If p varies directly as q and p is equal to 282, when q = 5.1. If q = 6.8, then what is p?
- **4.** Five spraying machines working together can finish painting a house in 36 minutes. If 2 machines break down, then in how much time can the remaining machines complete the job?
- **5.** A contractor employed 560 men to complete a project in 7 months. If he wishes to complete the same project in 5 months, how many more men should he employ?
- 6. Parth can do a piece of work in 20 days and Sachin can do the same work in 15 days. They work together for 5 days and then Sachin leaves. In how many days will Parth alone finish the remaining work?
- 7. A train 150 m long is running at a speed of 50 km/h. How long will it take to cross a telegraph post?
- **8.** A and B can do a piece of work in 40 days and 45 days respectively. They work together for 10 days and then B leaves. How many more days will A need to finish the remaining work?

Multiple C (Task Fo

Multiple Choice Questions (MCQs)

(Task For Formative Assessment)

1.	If $A: B = \frac{2}{3}$ and $B: C = \frac{4}{9}$, then $A: C$ is:			
	(a) 3:4	(b)	2:9	
	(c) 8:27	(d)	1:2	
2.	If 5 men can build a wall in 12 days, how ma	any men	can build it in 10 days?	
	(a) 8 men	(b)	7 men	
	(c) 6 men	(d)	4 men	
3.	Given two quantities x and y . If an incre	ease in x	causes a proportionate decrease in y	(and
	vice-versa) such that their product remains c	onstant tl	nen x and y are said to be :	
	(a) independent	(b)	directly related	
	(c) inversely related	(d)	none of these	
4.	If $a = 4b$, then a and b vary with	h each ot	her.	
	(a) inversely	(b)	directly	
	(c) insufficient information	(d)	none of these	
5.	x and y vary directly such that $x = 6$ and $y =$	18. If $y =$	15, the value of x is :	
	(a) 3	(b)	4	
	(c) 5	(d)	10	$\overline{\Box}$



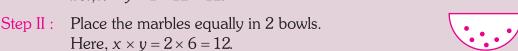
Aim: To develop the concept of inverse variation by activity method.

Materials required: 6 bowls and 12 marbles.

Process: Consider the number of bowls as 'x' and the number of marbles as 'y'.

Step I: Take one bowl and place 12 marbles in it. So, you have, Number of bowls \times Number of marbles = $1 \times 12 = 12$

i.e., $x \times y = 1 \times 12 = 12$.



Step III : Place the 12 marbles equally in 3 bowls. Here, $x \times y = 3 \times 4 = 12$.



Step IV: Place the 12 marbles equally in 4 bowls. Here, $x \times y = 4 \times 3 = 12$.



Step V: Place the 12 marbles equally in 6 bowls. Here, $x \times y = 6 \times 2 = 12$.



Step VI: What do you observe? As the number of bowls increases, the number of marbles placed in it decreases.

Number of bowls (x)	Number of marbles in each bowl (y)	Total number of marbles (k)
1	12	12
2	6	12
3	4	12
4	3	12
5	2	12

Step VII: In each case, the product $x \times y = \text{constant}$.

This means when x increases, y decreases and vice versa.

This shows that the value of x and y are in inverse variation.



Understanding Quadrilaterals

Sequence of the Chapter

- Polygons
- ✓ Convex And Concave Polygons
- ✓ Classification Of Polygons
- ✓ Regular And Irregular Polygons
- Quadrilaterals
- ✓ Convex And Concave Quadrilaterals
- ✓ Angle Sum Property Of A Quadrilateral



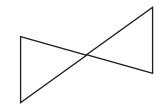
A **plane surface** is an unobstructed two-dimensional shape like a sheet of paper. If a number of points on a plane surface are joined without breaking or retracing, the curve so formed is called a **plane curve**.

A plane curve is a curve whose every point is also a point of the plane. All the points of the curve lie on the plane.

- ★ When no point of the curve is touched twice, we call the curve as a **simple curve**. In other words, a simple curve does not cross itself at any point.
- ★ A curve is said to be **closed** if it begins and ends at the same point. In other words, a closed curve joins up itself and no beginning and ending can be identified.



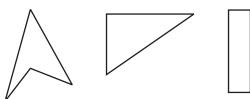
Simple and closed curve



Not simple but closed curve



Polygon is a closed curve made up of three or more line segments.



These closed curves are polygons.

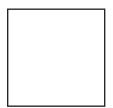


These closed curves are not polygons.

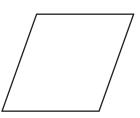
CONVEX AND CONCAVE POLYGONS

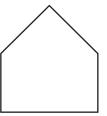
Convex polygon is a simple polygon, where :

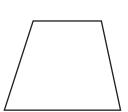
- \star each interior angle is less than 180°.
- ★ each line segment between two non-adjacent vertices are in or on the boundary of polygon.





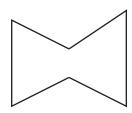


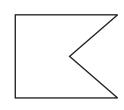


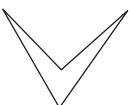


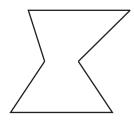
Concave polygon is a polygon, that is not convex. It has:

 \star at least one interior angle greater than 180°.











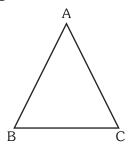
Polygons are classified according to the number of sides (or vertices) they have.

Figure	Number of sides or vertices	Classification
A IIII C	3	Triangle
	4	Quadrilateral
	5	Pentagon

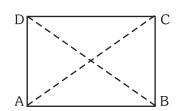
6	Hexagon
7	Septagon
8	Octagon
9	Nonagon
10	Decagon

DIAGONALS

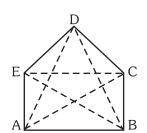
A diagonal of a polygon is a line segment that connects two non-consecutive vertices of a polygon.



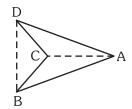
A triangle has no diagonals.



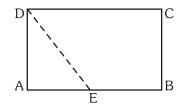
AC and BD are the diagonals of rectangle ABCD.



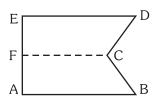
AC, AD, BD, BE and CE are the diagonals of the pentagon ABCDE.



AC and BD are diagonals.



DE is not a diagonal.

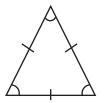


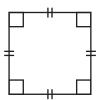
CF is not a diagonal.



REGULAR AND IRREGULAR POLYGONS

In a **regular polygon**, all sides are of the same length (equilateral) and all angles are of the same measure (equiangular).

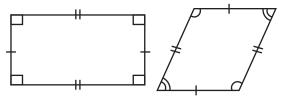






- \blacksquare The sum of the three angles of a triangle is 180° or equal to 2 right angles.
- The sum of measure of all exterior angles of any polygon is 360°.

A polygon which is not regular is an **irregular polygon**.



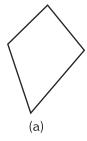


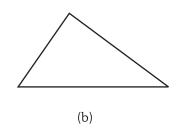
- 1. Write the names of the polygons having:
 - (a) 3 sides
- (b) 6 sides
- (c) 7 sides
- (d) 9 sides
- 2. How many diagonals do each of the following have?
 - (a) A hexagon

(b) A regular pentagon

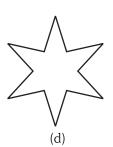
(c) A convex quadrilateral

- (d) A triangle
- 3. Which of the following figures are polygons?









- **4.** Draw a rough diagram to show:
- (a) open curve
- (b) closed curve.
- 5. Draw two examples of each of the following:
 - (a) Curves that are closed but not simple.
 - (b) Curves that are both closed and simple.
 - (c) Curves that are neither simple nor closed.

QUADRILATERALS

Quadrilateral is a simple closed plane figure formed by four line segments which do not intersect except at their end points.

In the adjoining figure, *ABCD* is a quadrilateral. It has:

 \rightarrow four vertices - A, B, C and D

 \rightarrow four sides $-\overline{AB}, \overline{BC}, \overline{CD}$ and \overline{DA}

 \rightarrow four angles - $\angle A$, $\angle B$, $\angle C$ and $\angle D$

 \rightarrow two diagonals - AC and BD

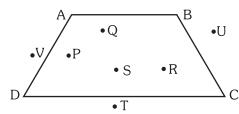
Two sides with a common vertex are **adjacent sides**. For example, sides AB and BC, BC and CD, CD and DA, DA and AB.

Two sides with no common vertex are **opposite sides**. For example, sides AB and DC, AD and BC.

A pair of interior angles with a common arm are **adjacent angles**. For example, $\angle DAB$ and $\angle ABC$, $\angle ABC$ and $\angle BCD$, $\angle BCD$ and $\angle CDA$, $\angle CDA$ and $\angle DAB$. A pair of interior angles with no common arm are **opposite angles**. For example, $\angle DAB$ and $\angle BCD$, $\angle ABC$ and $\angle CDA$.

All points lying inside the quadrilateral are **interior points** of the quadrilateral. *For example*, points P, Q, R, S.

All points lying outside the quadrilateral are **exterior points** of the quadrilateral. For example, points T, U, V.



The interior of a quadrilateral together with its boundary is the **quadrilateral region**.

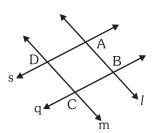


CONVEX AND CONCAVE QUADRILATERALS

Convex Quadrilateral

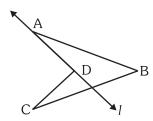
Consider the quadrilateral ABCD as shown alongside. l is the line containing vertices A and B. q is the line containing the vertices B and C. Line m contains the vertices C and D. Line S contains the vertices S and S and S lie on the same side of S and S lie on the same side of S.

Such a quadrilateral, in which a line containing two of its vertices has the remaining two vertices on the same side is called a **convex quadrilateral**.



Concave Quadrilateral

Consider the quadrilateral ABCD as shown alongside. Line l contains the vertices A and D and the remaining two vertices B and C lie on the opposite side of l. Such a quadrilateral is called **concave quadrilateral**.



Properties Of Convex And Concave Quadrilateral

1. In a convex quadrilateral, a line segment joining any two points in the interior of the quadrilateral lies completely inside the quadrilateral.

In a concave quadrilateral, at least for one pair of interior points, the line segment joining these points does not lie completely inside the quadrilateral.

- 2. In a convex quadrilateral, all angles are less than 180°. In a concave quadrilateral, one angle is more than 180°.
- 3. In a convex quadrilateral, both the diagonals lie in the interior of the quadrilateral. In a concave quadrilateral, one diagonal lies completely inside the quadrilateral and one lies completely outside the quadrilateral.

ANGLE SUM PROPERTY OF A QUADRILATERAL

"The sum of all the angles of a quadrilateral is equal to 360°."

Proof: In the adjoining figure, ABCD is a quadrilateral and diagonal AC divides it into two triangles, i. e., $\triangle ABC$ and $\triangle ACD$.

In the adjoining figure,
$$ABCD$$
 is a quadrilateral and C divides it into two triangles, i. e., $\triangle ABC$ and $\triangle ACD$.

In $\triangle ACD$,

 $\angle ADC + \angle DAC + \angle ACD = 180^{\circ}$

..... (ii)

Adding (i) and (ii), we get

In $\triangle ACD$,

In $\triangle ABC$,

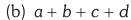
$$\angle ADC + \angle ABC + (\angle DAC + \angle BAC) + (\angle ACD + \angle BCA) = 180^{\circ} + 180^{\circ}$$

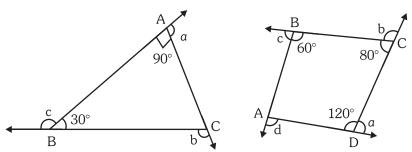
 $\angle ADC + \angle ABC + \angle DAB + \angle BCD = 360^{\circ}$ or,

 $\angle ABC + \angle BAC + \angle BCA = 180^{\circ}$

Example 1: Look at the figures given below and find:

(a)
$$a+b+c$$





Solution:

(a) Since a, b and c are the exterior angles of $\triangle ABC$, so

$$a + b + c = 360^{\circ}$$

 $90^{\circ} + a = 180^{\circ}$
 $\Rightarrow a = 90^{\circ}$
 $c + 30^{\circ} = 180^{\circ}$
 $\Rightarrow c = 150^{\circ}$
In $\triangle ABC$,
 $\angle A + \angle B + \angle C = 180^{\circ}$ (Sum of the angles of a \triangle is 180° .)
 $90^{\circ} + 30^{\circ} + \angle C = 180^{\circ}$
 $\Rightarrow \angle C = 180^{\circ} - 120^{\circ} = 60^{\circ}$
Now, $b + \angle C = 180^{\circ}$
So, $b = 180^{\circ} - 60^{\circ} = 120^{\circ}$

Therefore, $a + b + c = 90^{\circ} + 150^{\circ} + 120^{\circ} = 360^{\circ}$.

(b) Since a, b, c and d are the exterior angles of the quadrilateral ABCD, so $a+b+c+d=360^{\circ}$

In quadrilateral ABCD,

$$\angle A + \angle B + \angle C + \angle D = 360^{\circ}$$

(Sum of the four angles of a quadrilateral is 360°.)

$$\angle A + 60^{\circ} + 80^{\circ} + 120^{\circ} = 360^{\circ}$$

$$\Rightarrow$$
 $\angle A + 260^{\circ} = 360^{\circ}$

$$\Rightarrow$$
 $\angle A = 360^{\circ} - 260^{\circ} = 100^{\circ}$

Now,
$$d = 180^{\circ} - \angle A = 180^{\circ} - 100^{\circ} = 80^{\circ}$$

$$c = 180^{\circ} - \angle B = 180^{\circ} - 60^{\circ} = 120^{\circ}$$

 $b = 180^{\circ} - \angle C = 180^{\circ} - 80^{\circ} = 100^{\circ}$

$$a = 180^{\circ} - \angle D = 180^{\circ} - 120^{\circ} = 60^{\circ}$$

Therefore, $a + b + c + d = 60^{\circ} + 100^{\circ} + 120^{\circ} + 80^{\circ} = 360^{\circ}$.

- **Example 2:** The measure of two adjacent angles of a quadrilateral are 100° and 50° and other two angles are equal. Find the measure of each of equal angles.
- **Solution**: Suppose *ABCD* is the quadrilateral such that $\angle A = 100^{\circ}$, $\angle B = 50^{\circ}$.

Let
$$\angle C = \angle D = x$$
 (say)

Now, by angle sum property, we have

$$\angle A + \angle B + \angle C + \angle D = 360^{\circ}$$

$$\Rightarrow 100^{\circ} + 50^{\circ} + x + x = 360^{\circ}$$

$$\Rightarrow 150^{\circ} + 2x = 360^{\circ}$$

$$\Rightarrow 2x = 360^{\circ} - 150^{\circ}$$

$$\Rightarrow 2x = 210^{\circ}$$

$$\Rightarrow x = 105^{\circ}$$

$$\therefore$$
 $\angle C = \angle D = 105^{\circ}$.

- **Example 3**: The angles of a quadrilateral are in the ratio 1:2:2:4. Find the measure of each angle.
- **Solution**: Let the ratio constant be x. Thus, the measure of the given angles will be $(1x)^{\circ}$, $(2x)^{\circ}$, $(2x)^{\circ}$ and $(4x)^{\circ}$.

As we know that the sum of the angles of a quadrilateral is 360° , so

$$(1x)^{\circ} + (2x)^{\circ} + (2x)^{\circ} + (4x)^{\circ} = 360^{\circ}$$

$$\Rightarrow$$
 $(9x)^{\circ} = 360^{\circ}$

$$\Rightarrow x^{\circ} = \frac{360^{\circ}}{9}$$

$$\Rightarrow x^{\circ} = 40^{\circ}$$

Hence, the angles are:

$$1x = 1 \times 40^\circ = 40^\circ,$$

$$2x = 2 \times 40^{\circ} = 80^{\circ}$$

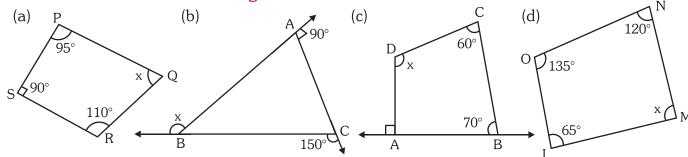
$$2x = 2 \times 40^{\circ} = 80^{\circ}$$

$$4x = 4 \times 40^{\circ} = 160^{\circ}$$

Hence, the angles are 40°, 80°, 80° and 160°.



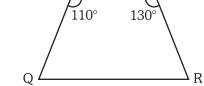
1. Find the value of x in each figure:



2. Two angles of a quadrilateral are 75° and 140°. The other two angles are equal. Find the other two angles.

3. Three angles of a quadrilateral are in the ratio 3:5:7. The difference of the least and the greatest of these angles is 76° . Find all the four angles of the quadrilateral.

4. PQRS is a quadrilateral in which $PS \parallel QR$. If $\angle P = 110^{\circ}$ and $\angle S = 130^{\circ}$, what are the measures of the other two angles?



- **5.** The three angles of a quadrilateral are 95°, 105° and 45°. Find its fourth angle.
- **6.** *PQRS* is a quadrilateral with $\angle P = \angle Q$ and $\angle R = \angle S$. If $\angle P = 2\angle R$, find the measure of each angle.
- **7.** Four angles of a quadrilateral are in the ratio 1:2:3:4. Find the angles.
- **8.** If three angles of a quadrilateral are 90° , 60° and 90° , find the fourth angle of the quadrilateral.

TRICKY CHALLENGE!

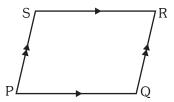
The sum of the interior angles of a polygon is three times the sum of its exterior angles. Determine the number of sides of the polygon.



TYPES OF QUADRILATERALS

Parallelogram

A quadrilateral in which two pairs of opposite sides are parallel and equal, i. e., $PQ \parallel SR$ and $PS \parallel QR$, and PQ = SR and PS = QR, is called a parallelogram.



Properties of a Parallelogram:

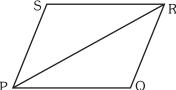
1. Opposite sides of a parallelogram are equal.

Let us consider a parallelogram PQRS in which $PQ \parallel RS$ and $QR \parallel PS$. Join P to R.

Now, in $\triangle PQR$ and $\triangle RSP$,

 $\angle RPQ = \angle PRS$ (Alternate angles)

 $\angle PRQ = \angle RPS$ (Alternate angles)



PR = PR (Common)

 $\Delta PQR \cong \Delta RSP$ (ASA congruency)

Hence, PQ = RS and QR = PS (By c.p.c.t.)

Thus, opposite sides of a parallelogram are equal.

2. Opposite angles of a parallelogram are equal.

Let us consider a parallelogram PQRS in which $PQ \parallel RS$ and $QR \parallel PS$.

Now, $PQ \parallel SR$ and PS is transversal.

$$\therefore \qquad \angle S + \angle P = 180^{\circ} \qquad \qquad \dots \dots \dots (i)$$

(The sum of the measure of interior angles on the same side of a transversal is 180° .)

Again $PS \parallel QR$ and PQ is a transversal.

Therefore, $\angle P + \angle Q = 180^{\circ}$ (ii)

From (i) and (ii), we have

$$\angle S + \angle P = \angle P + \angle Q$$

or, $\angle S = \angle Q$

Similarly, we can prove $\angle P = \angle R$.

Thus, opposite angles of a parallelogram are equal.

3. Diagonals of a parallelogram bisect each other.

Let us consider a parallelogram PQRS in which $PQ \parallel RS$ and $PS \parallel QR$.

Join P to R and Q to S. Let they intersect at O.

Now, in $\triangle POQ$ and $\triangle ROS$,

$$\angle SRO = \angle OPQ$$
 (Alternate angles) $\angle RSO = \angle OQP$ (Alternate angles)

$$PQ = RS$$
 (Opposite sides of a parallelogram) P

So,
$$\Delta POQ \cong \Delta ROS$$
 (ASA congruency)

Hence, PO = OR

and,
$$QO = OS$$
 (By c.p.c.t.)

Thus, diagonals of a parallelogram bisect each other.

4. Each diagonal of a parallelogram divides it into two congruent triangles.

Let us consider a parallelogram PQRS where $PQ \parallel RS$ and $PS \parallel QR$. Join P to R.

Diagonal PR divides the parallelogram into two triangles, i.e., ΔPQR and ΔRSP .

Now, in $\triangle PQR$ and $\triangle RSP$,

$$\angle SRP = \angle QPR$$
 (Alternate angles)
 $\angle SPR = \angle PRQ$ (Alternate angles)

So,
$$\angle Q = \angle S$$
 (Angle sum property)

$$PQ = RS$$
 (Opposite sides of a parallelogram)

Hence,
$$\Delta PQR \cong \Delta RSP$$
 (ASA congruency)

Similarly, by drawing the other diagonals QS, we can prove that $\Delta PQS \cong \Delta RSQ$.

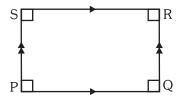
Thus, each diagonal of a parallelogram divides it into two congruent triangles.

Rectangle

A quadrilateral whose each angle is equal to 90° , is called a rectangle. In the adjoining figure, quadrilateral *PQRS* is a rectangle because

$$PQ \parallel SR$$
 and $PS \parallel QR$

and,
$$\angle P = \angle Q = \angle R = \angle S = 90^{\circ}$$
.



Properties of a Rectangle:

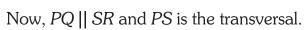
1. Each angle of a rectangle is a right angle.

Let us consider a rectangle *PQRS* in which $\angle P = 90^{\circ}$.

Also, PQRS is a parallelogram.

Therefore,
$$PQ = SR$$
, $QR = PS$ and $\angle P = \angle R$, $\angle Q = \angle S$

Here,
$$\angle R = 90^{\circ}$$



Therefore,

$$\angle P + \angle S = 180^{\circ}$$

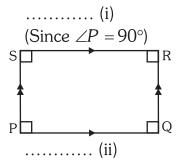
$$90^{\circ} + \angle S = 180^{\circ}$$

$$\angle S = 180^{\circ} - 90^{\circ} = 90^{\circ}$$

But
$$\angle Q = \angle S$$

So,
$$\angle Q = 90^{\circ}$$

Hence,
$$\angle P = \angle Q = \angle R = \angle S = 90^{\circ}$$



[From (i), (ii) and (iii)]

Thus, each angle of a rectangle is a right angle.

2. Diagonals of a rectangle are equal and bisect each other.

Let us consider a rectangle *PQRS* with diagonals *PR* and *QS*. As *PQRS* is a rectangle,

and
$$\angle P = \angle Q = \angle R = \angle S = 90^{\circ}$$

Now, in rectangle PQRS, since PQ | SR and QR | PS

- .. PQRS is a parallelogram.
- \Rightarrow QS and PR bisect each other.

(Because diagonals of parallelogram bisect each other.)

Now, considering ΔSPQ and ΔRQP , we have

$$PS = QR$$

(Opposite sides of parallelogram *PQRS*)

$$PQ = PQ$$

(Common)

$$\angle SPQ = \angle RQP$$

(90° each)

$$\therefore \quad \Delta SPQ \cong \Delta RQP$$

(By SAS congruency)

$$\Rightarrow$$
 $QS = PR$

(By c.p.c.t)

Thus, diagonals of a rectangle are equal and bisect each other.

Example 4: Two adjacent angles of a parallelogram are in the ratio 1:2. Find the measures of all the angles of the parallelogram.

Solution: Suppose PQRS is a parallelogram and measures of

 $\angle P$ and $\angle Q$ are in the ratio 1:2.

Let $\angle P = x$, then $\angle Q = 2x$.

Now, $PS \parallel QR$ and PQ is transversal.

So,
$$\angle P + \angle Q = 180^{\circ}$$

(: Sum of the interior angles on one side of parallel line is 180°.)

$$\Rightarrow$$
 $x + 2x = 180^{\circ} \Rightarrow 3x = 180^{\circ} \Rightarrow x = 60^{\circ}$

$$\therefore$$
 $\angle P = 60^{\circ} \text{ and } \angle Q = 2 \times 60^{\circ} = 120^{\circ}$

Again, opposite angles of a parallelogram are equal.

$$\therefore$$
 $\angle R = \angle P = 60^{\circ}$

and,
$$\angle S = \angle Q = 120^{\circ}$$
.

Example 5: The diagonals of a rectangle PQRS intersect each other at O. If $\angle POS = 40^{\circ}$,

find $\angle ROQ$ and $\angle ORQ$.

Solution: In the given rectangle *PQRS*,

 $\angle ROQ = \angle POS = 40^{\circ}$ (Vertically opposite angles)

Therefore, $\angle ROQ = 40^{\circ}$

In $\triangle ORQ$, OR = OQ

(Diagonals of a rectangle are equal and bisect each other.)

So, $\angle ORQ = \angle OQR = x$ (say) (Angles opposite to equal sides are equal.)

Therefore, in $\triangle ORQ$, we have

$$\angle ROQ + \angle OQR + \angle ORQ = 180^{\circ}$$

$$40^{\circ} + x + x = 180^{\circ}$$

$$\Rightarrow$$
 $2x = 180^{\circ} - 40^{\circ} = 140^{\circ}$

$$\Rightarrow x = \frac{140^{\circ}}{2} = 70^{\circ}$$

Hence, $\angle ROQ = 40^{\circ}$ and $\angle ORQ = 70^{\circ}$.

Example 6: Find angles x, y, z in the adjoining figure.

Solution: We know adjacent angles of a parallelogram are supplementary.

$$\therefore \qquad \angle A + \angle D = x + 50^{\circ} = 180^{\circ}$$

$$\Rightarrow \qquad x = 180^{\circ} - 50^{\circ} = 130^{\circ}$$

And, opposite angles of a parallelogram are equal.

$$\therefore \qquad \angle C + \angle A = z = x = 130^{\circ}$$

Also, since alternate interior angles are equal, $y = z = 130^{\circ}$.

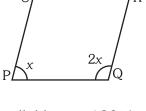
Thus,
$$x = 130^{\circ}$$
, $y = 130^{\circ}$ and $z = 130^{\circ}$.

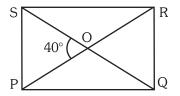


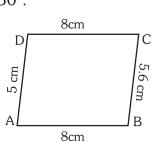
(a)
$$\angle B + \angle D = 180^{\circ}$$
?

(b)
$$AB = DC = 8$$
 cm, and $AD = 5$ cm and $BC = 5.6$ cm?

(c)
$$\angle A = 60^{\circ}$$
 and $\angle C = 65^{\circ}$?







Solution:

(a)
$$\angle B + \angle D = 180^{\circ}$$

In a parallelogram, opposite angles are equal.

$$\Rightarrow$$
 $\angle B = \angle D$

Since
$$\angle B + \angle D = 180^{\circ}$$

$$\Rightarrow$$
 $\angle B = \angle D = 90^{\circ}$

The parallelogram is a rectangle.

So, quadrilateral *ABCD* will be a parallelogram only if it is a rectangle otherwise not.

- (b) AB = DC = 8 cm, AD = 5 cm, BC = 5.6 cm No, quadrilateral ABCD cannot be a parallelogram with these conditions, because $AD \neq BC$.
- (c) $\angle A = 60^\circ$ and $\angle C = 65^\circ$ No, quadrilateral *ABCD* cannot be a parallelogram with this condition, because $\angle A \neq \angle C$.

Example 8: The adjacent sides of a parallelogram differ by 20 cm. If its perimeter is 140 cm, find its sides.

Solution: Let the two adjacent sides of the parallelogram be x cm and (x + 20) cm. It is given that the perimeter of the parallelogram is 140 cm, therefore

$$2 \times \{x + (x + 20)\} = 140$$

$$\Rightarrow 2x + 20 = \frac{140}{2} = 70$$

$$\Rightarrow 2x = 70 - 20$$

$$\Rightarrow 2x = 50$$

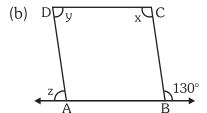
$$\Rightarrow x = \frac{50}{2} = 25$$

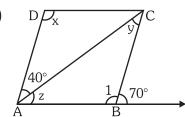
Thus, the sides of the parallelogram are 25 cm, 45 (25 + 20) cm, 25 cm, 45 (25 + 20) cm.



1. Find angles x, y, z in each of the following parallelograms:

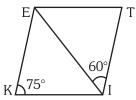
(a) D_{x} y O_{x} 105° B_{x}



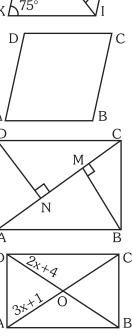


- 2. The measures of two adjacent angles of a parallelogram are in the ratio 3:2. Find the measure of each of the angles of parallelogram.
- **3.** The longer side of a parallelogram is 8.4 cm and the shorter side is half of the longer side. Find the perimeter of the parallelogram.

4. KITE is a parallelogram in which $\angle EKI = 75^{\circ}$ and $\angle EIT = 60^{\circ}$. Calculate the measures of $\angle TEI$ and $\angle KEI$.



- 5. In each of the following cases, can there be parallelogram ABCD in which:
 - (a) $\angle A = 70^{\circ}$ and $\angle B = 110^{\circ}$?
 - (b) AB = DC = 7 cm, AD = 5 cm, BC = 5.2 cm?
 - (c) $\angle B = 80^{\circ}$ and $\angle D = 75^{\circ}$?
- In the adjoining figure, ABCD is a rectangle. BM and DN are 6. perpendiculars from B and D to AC. Prove that $\triangle BMC \cong \triangle DNA$.

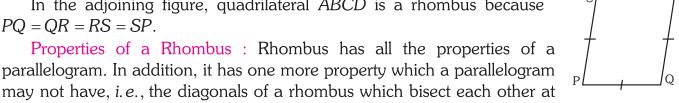


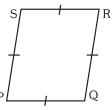
- **7.** ABCD is a rectangle whose diagonals meet at O. Find x, if OA = 3x + 1 and OD = 2x + 4.
- The adjacent angles of a parallelogram are $(3x-12)^{\circ}$ and $(52+2x)^{\circ}$. Find the angles of the 8. parallelogram.

Rhombus

A quadrilateral whose all sides are equal, is called a rhombus.

In the adjoining figure, quadrilateral ABCD is a rhombus because PQ = QR = RS = SP.





Let us consider a rhombus PQRS with diagonals PR and QS.

In $\triangle POQ$ and $\triangle QOR$,

$$PQ = QR$$
 (Sides of a rhombus)

$$QO = QO$$
 (Common)

$$PO = OR$$
 (Diagonals bisect each other)

Therefore,

right angles.

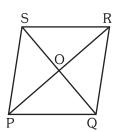
$$\Delta POQ \cong \Delta QOR$$
 (By SSS congruency)

Hence,
$$\angle POQ = \angle QOR$$
 (By c.p.c.t.)
But $\angle POQ + \angle QOR = 180^{\circ}$ (Linear pair)

So,
$$2\angle POQ = 180^{\circ}$$

$$\Rightarrow$$
 $\angle POQ = 90^{\circ} = \angle QOR$

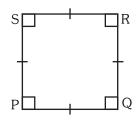
Thus, diagonals bisect each other at right angles.



Square

A quadrilateral whose all sides are equal and each angle is equal to 90° , is called a square.

In the adjoining figure, PQRS is a square, in which PQ = QR = RS = SP and $\angle P = \angle Q = \angle R = \angle S = 90^{\circ}$.



Properties of a Square: A square is a rhombus with each of its angles as 90° . Hence, it has all the properties of a rhombus, *i.e.*,

- ★ All the sides are equal.
- ★ Opposite sides are parallel.
- ★ All the angles are equal.
- ★ Diagonals bisect each other at right angles.

In addition to these properties, it has one more property which a rhombus does not have, *i.e.*, the diagonals of a square are equal.

Let us consider a square PQRS with diagonals PR and QS.

In $\triangle PQS$ and $\triangle QPR$,

$$PS = QR$$

(Opposite sides of a square)

$$PQ = PQ$$

(Common)

$$\angle SPQ = \angle PQR = 90^{\circ}$$

Therefore,
$$\triangle PQS \cong \triangle QPR$$

(By SAS congruency)

Hence,
$$PR = QS$$

(By c.p.c.t.)

Thus, diagonals of a square are equal.

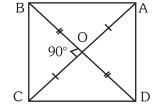
Example 9: The diagonals of a rhombus ABCD intersect at O. If AC = 8 cm and BD = 6

cm, find the length of its sides.

Solution: Since, AC = 8 cm

$$\Rightarrow OA = OC = 4 \text{ cm}$$

(: Diagonals of a rhombus bisect each other at right angle.)



And,
$$BD = 6 \text{ cm}$$

$$\Rightarrow$$
 $OB = 3 \text{ cm}$

Also,
$$\angle BOC = 90^{\circ}$$

Hence, $\triangle BOC$ is a right-angled triangle.

$$BC^2 = OB^2 + OC^2$$
 (Using Pythagoras theorem)

$$\Rightarrow$$
 $BC^2 = 4^2 + 3^2 = 16 + 9$

$$\Rightarrow BC^2 = 25$$

$$\Rightarrow$$
 $BC^2 = 5^2$

$$\Rightarrow$$
 BC = 5 cm

Hence, length of each side of the rhombus is 5 cm.

Example 10: AC is the diagonal of a square ABCD. Find the measure of $\angle ACB$.

Solution: We know that all sides of a square are equal.
$$\therefore AB = BC$$

$$\angle BCA = \angle BAC = x$$
 (let) (Angles opposite to equal sides are equal.) In $\triangle ABC$,

$$\angle BAC + \angle ABC + \angle BCA = 180^{\circ}$$
 (Angle sum property of a triangle)

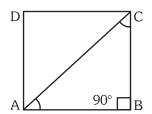
$$\Rightarrow x + 90^{\circ} + x = 180^{\circ}$$

$$\Rightarrow 2x + 90^{\circ} = 180^{\circ}$$

$$\Rightarrow 2x = 180^{\circ} - 90^{\circ} = 90^{\circ}$$

$$\Rightarrow x = \frac{90^{\circ}}{2} = 45^{\circ}$$

Therefore, $\angle ACB = 45^{\circ}$.



(Using Pythagoras theorem)

(:: AB = BC = CD = DA)

Example 11: Length of a diagonal of a square is 10 cm. Find the length of each side of the

square.

Solution: Given, AC = 10 cm

Let
$$AB = x$$

Since,
$$\angle B = 90^{\circ}$$

 \therefore $\triangle ABC$ is a right-angled triangle.

In
$$\triangle ABC$$
, $AC^2 = AB^2 + BC^2$

$$\Rightarrow 10^2 = x^2 + x^2$$

$$\Rightarrow 100 = 2x^2$$

Or,
$$2x^2 = 100$$

$$\Rightarrow x^2 = \frac{100}{2} = 50$$

$$\Rightarrow x = \sqrt{50} = \sqrt{2 \times 5 \times 5} = 5\sqrt{2} = 7.07 \text{ (approx .)}$$

Since all sides are equal, so

$$AB = BC = CD = DA = 7.07$$
 cm.

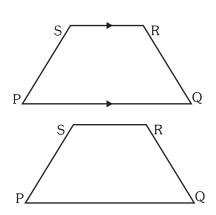


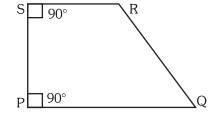
A quadrilateral in which one pair of opposite sides are parallel, is called a trapezium.

In quadrilateral PQRS, $PQ \mid\mid RS$. So, PQRS is a trapezium.

Types of Trapezium:

1. A trapezium in which the non-parallel sides are equal is called an **isosceles trapezium**. In trapezium PQRS, $PQ \parallel SR$ and PS = QR. So, it is an isosceles trapezium.





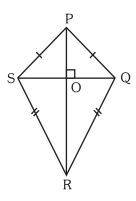
2. A trapezium that has two right angles is a **right trapezium**. In trapezium PQRS, $\angle P = \angle S = 90^{\circ}$. So, it is a right trapezium.

Kite

A quadrilateral with two pairs of adjacent sides of the same length is called a kite.

Properties of a Kite:

- 1. None of the sides are parallel.
- 2. Two pairs of adjacent sides are equal.
- 3. The diagonals intersect at right angles ($PR \perp SQ$).
- 4. The longer diagonal bisects the shorter one.

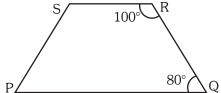


Example 12: Explain how PQRS is a trapezium. Which of

its two sides are parallel?

Solution: In quadrilateral *PQRS*,

$$\angle Q + \angle R = 80^{\circ} + 100^{\circ} = 180^{\circ}$$



But these are interior angles on the same side of the transversal RQ between the lines SR and PQ.

$$\Rightarrow$$
 SR || PQ

(One pair of opposite sides is parallel.)

Hence, *PQRS* is a trapezium.

Example 13: In the given kite ABCD, $\angle ABC = 70^{\circ}$ and $\angle ADC = 80^{\circ}$,

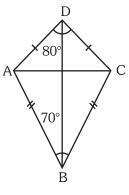
find the remaining two angles.

Solution: Since, adjacent sides of kite are equal, i.e., AD = DC and

$$AB = BC$$
.

$$\angle DCA = \angle DAC = x$$
 (let)

And, $\angle BCA = \angle BAC = y$ (let) (Angles opposite to equal sides are also equal.)



In $\triangle ADC$,

$$\angle ADC + \angle DAC + \angle DCA = 180^{\circ}$$
 (Angle sum property of a triangle)

$$\Rightarrow$$
 80° + x + x = 180°

$$\Rightarrow 2x = 180^{\circ} - 80^{\circ} = 100^{\circ}$$

$$\Rightarrow \qquad x = \frac{100^{\circ}}{2}$$

$$\Rightarrow$$
 $x = 50^{\circ}$

Therefore,
$$\angle DCA = \angle DAC = 50^{\circ}$$

In
$$\triangle ABC$$
,

$$\angle ABC + \angle BCA + \angle BAC = 180^{\circ}$$
 (Angle sum property of a triangle)

$$\Rightarrow$$
 70° + y + y = 180°

$$\Rightarrow 2y = 180^{\circ} - 70^{\circ} = 110^{\circ}$$

$$\Rightarrow y = \frac{110^{\circ}}{2}$$

$$\Rightarrow y = 55^{\circ}$$

Therefore,
$$\angle BCA = \angle BAC = 55^{\circ}$$

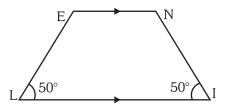
Now, $\angle DAB = \angle DAC + \angle BAC = 50^{\circ} + 55^{\circ} = 105^{\circ}$

And, $\angle DCB = \angle DCA + \angle BCA = 50^{\circ} + 55^{\circ} = 105^{\circ}$

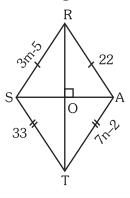
Therefore, remaining two angles are 105° each.



1. LINE is a trapezium in which LI || EN. If $\angle L = 50^{\circ}$ and $\angle I = 50^{\circ}$, find the other two angles.



- 2. Prove that each diagonal of a rhombus bisects the angle through which it passes.
- 3. In a square, one diagonal is 16 cm. Find the side of the square.
- PQ is a rhombus in which the altitude from S to side PQ bisects side PQ. Find the angles of the rhombus.
- ABCD is a rhombus whose diagonals intersect at O. If AB = 10 cm and BD = 16 cm, find the length 5. of diagonal AC.
- Two adjacent angles of a rhombus are in the ratio 3:6. Find the measure of all the angles. 6.
- STAR is a kite, in which diagonals SA and RT intersect at point O. Find the values of m and n.
- 8. In a trapezium, two adjacent angles are 50° and 60°. Find the remaining two angles.



SUMMARY OF THE CHAPTER

- Polygon is a closed curve made up of three or more line segments.
- Quadrilateral is a simple closed plane figure formed by four line segments which do not intersect except at their end points.
- The sum of all the angles of a quadrilateral is equal to 360°.
- A quadrilateral in which two pairs of opposite sides are parallel and equal, is called a parallelogram. 回
- A quadrilateral whose each angle is equal to 90°, is called a rectangle.
- A quadrilateral whose all sides are equal, is called a rhombus.
- A quadrilateral whose all sides are equal and each angle is equal to 90°, is called a square.
- A quadrilateral in which one pair of opposite sides are parallel, is called a trapezium. 回
- A quadrilateral with two pairs of adjacent sides of the same length, is called a kite.

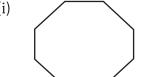


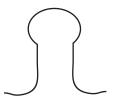
Review Of The Chapter (Task For Summative Assessment)

- 1. Classify each of the figures on the basis of the following:
 - (a) Closed curve
- (b) Simple closed curve
- (c) Polygon

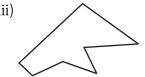
- (d) Convex polygon
- (e) Concave polygon
- (f) Not a curve



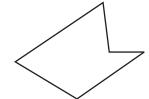




(iii)

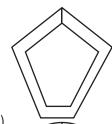


(iv)

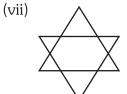


(v)

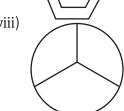
(ii)

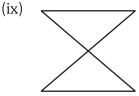




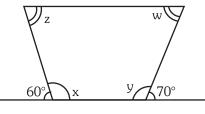


(viii)

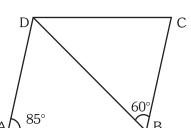




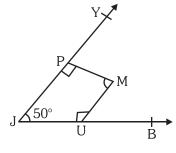
- 2. If the angles of a triangle are in the ratio 2:3:4, find the angles.
- 3. The angles of a quadrilateral are in the ratio 1:2:3:4. Find the measure of each angle.
- 4. Find the missing angles of the adjoining figure.



- In the adjoining figure, ABCD is a parallelogram. **5.** If $\angle DAB = 85^{\circ}$ and $\angle DBC = 60^{\circ}$, then calculate :
 - (a) ∠CDB
 - (b) ∠ABD



- The length of each diagonal of a quadrilateral is 7 cm. If its diagonals bisect each other at 90°, what **6.** special name will be given to this quadrilateral and why?
- The adjacent sides of a rectangle are in the ratio 5:3 and its perimeter is 32 cm. Find the lengths of 7. the sides.
- Perimeter of a kite is 80 cm and its one side is 20 cm. Find the other three sides. 8.
- If one of the diagonals of a rhombus is equal to one of its sides, find the angles of the rhombus. 9.
- In the given figure, find the measure of $\angle UMP$.



Multiple Choice Questions (MCQs) (Task For Formative Assessment)

1.	A figure that is both a rectangle and a rhombus is :						
	(a) kite	(b	o) rhombus				
	(c) rectangle	(d	l) square				
2.	A quadrilateral with two pairs of parallel sid						
	(a) kite	(b	o) parallelogram				
	(c) square	(d	l) rhombus				
3.	A figure in which the longer diagonal bisect	orter one is :					
	(a) kite	(b	o) parallelogram				
	(c) trapezium	(d	l) square				
4.	The diagonals of an isosceles trapezium are	e :					
	(a) equal	(b	o) unequal				
	(c) both (a) and (b)	(d	l) none of these				
5.	A regular polygon is:						
	(a) equilateral	(b	o) equiangular				
	(c) both (a) and (b)	(d	l) none of these				
6.	The bisectors of two adjacent angles of a parallelogram intersect at :						
	(a) 30°	(b	b) 45°				
	(c) 60°	(d	l) 90°				
7.	The quadrilateral formed by joining the mid-points of the sides of a quadrilateral is a :						
	(a) square	(b	o) rectangle				
	(c) parallelogram	(d	l) kite				
8.	The figure formed by the intersection of the angle bisectors of a parallelogram is always :						
	(a) a rhombus	(b	o) a rectangle				
	(c) a square	(d	l) any other parallelogram				
9.	In the case of a trapezium, the ratio of its angles taken in order cannot be :						
	(a) 1:2:3:4	(b) 7:13:2:8				
	(c) 6:3:4:2	(d	1) 4:5:2:3				
10.	In the adjoining figure, $ABCD$ is a parallelogram. The $\angle ABD$ is :						
			A B				
	(a) 35°	(b					
	(c) 80°	(d	d) 75°				
			$\sim 235^{\circ}$				



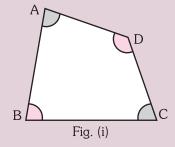
Aim: To verify practically that the sum of the angles of a quadrilateral is 360°.

Materials required: Drawing sheet, scale, marker pen, colour, a pair of scissors, glue stick.

Process:

Step I: On a drawing sheet draw two congruent quadrilaterals and name it as *ABCD*.

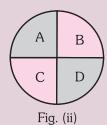
Step II: Cut out the angles from one of the quadrilaterals.



Step III: Arrange the cut-outs in such a way that vertex of each angle coincides at a point, see fig (ii).

Observation:

1. $\angle A + \angle B + \angle C + \angle D = 360^{\circ}$ (As sum of the angles at a point is 360°.)



2. Thus, sum of the angles of a quadrilateral is 360°.



Construction Of Quadrilaterals

Sequence of the Chapter

- ✓ Construction Of A Quadrilateral When Its Three Sides And Two Diagonals Are Given
- ✓ Construction Of A Quadrilateral When Its Four Sides And One Angle Are Given
- Construction Of A Quadrilateral When Its Three Angles And Two Included Sides Are Given
- ✓ Construction Of Special Types Of Quadrilaterals

INTRODUCTION

We already know that a quadrilateral is a closed figure. It has:

(a) four sides

(b) four angles

Therefore, it has eight parts. Now a question arises, do we need all the eight measurements to construct a quadrilateral?

No, we do not need to know the measures of all the eight parts to construct a quadrilateral. We need to know only five out of the eight parts.

In this chapter, we will learn to construct a convex quadrilateral with the following conditions:

- \star When the measures of 4 sides and 1 diagonal are given.
- ★ When the measures of 3 sides and 2 diagonals are given.
- \star When the measures of 4 sides and 1 enclosed angle are given.
- \star When the measures of 3 sides and 2 enclosed angles are given.
- \star When the measures of 2 sides and 3 enclosed angles are given.

CONSTRUCTION OF A QUADRILATERAL WHEN ITS FOUR SIDES AND ONE DIAGONAL ARE GIVEN

Example 1: Construct a quadrilateral PQRS with PQ = 3 cm, QR = 3.5 cm, RS = 4.2 cm,

PS = 3.4 cm and diagonal PR = 5 cm.

Solution: First draw a rough sketch of the figure to be

constructed.

Steps of Construction:

Step I: Draw diagonal PR = 5 cm.



5cm

Step II: With P as the centre and a radius $3.4~\mathrm{cm}$ draw an arc on one side of

PR.

Step III: With *R* as the centre and radius 4.2 cm, draw an arc intersecting the

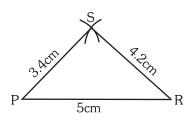
first arc at *S*.

Step IV: Join P to S and R to S.

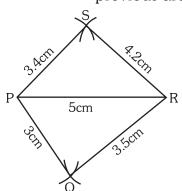
Step V: With P as the centre and a radius 3 cm,

draw an arc on the other side of PR.

Step VI: With R as the centre and a radius 3.5 cm, draw an arc intersecting the previous arc at Q.



5cm





Step VII: Join P to Q and R to Q.

The figure *PQRS*, thus drawn, is the required quadrilateral.

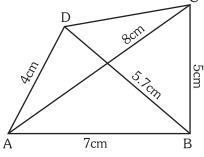
CONSTRUCTION OF A QUADRILATERAL WHEN ITS THREE SIDES AND TWO DIAGONALS ARE GIVEN

Example 2: Construct a quadrilateral ABCD, where AB = 7 cm, BC = 5 cm, DA = 4 cm,

AC = 8 cm and BD = 5.7 cm.

Solution: First draw a rough sketch of the figure to

be constructed.

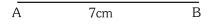


Steps of Construction:

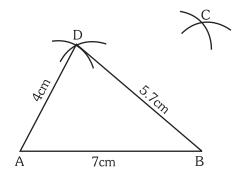
Step I: Draw AB = 7 cm.



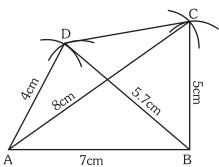
Step II: With *A* and *B* as centres, and radii 4 cm and 5.7 cm respectively, draw arcs intersecting each other at *D*.



Step III : Join A to D and B to D.



Step IV: Again, with A and B as centres, and radii 8 cm and 5 cm respectively, draw arcs intersecting each other at C.



Step V: Join D to C, A to C and B to C.

The figure ABCD, thus drawn, is the required quadrilateral.

CONSTRUCTION OF A QUADRILATERAL WHEN ITS FOUR SIDES AND ONE ANGLE ARE GIVEN

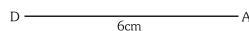
Example 3: Construct a quadrilateral ABCD such that AB = 5 cm, BC = 3 cm, CD = 10

cm, DA = 6 cm and $\angle D = 45^{\circ}$.

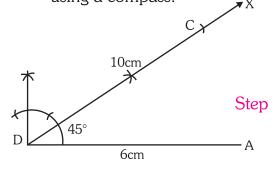
Solution: First draw a rough sketch of the figure to be constructed.

Steps of Construction:

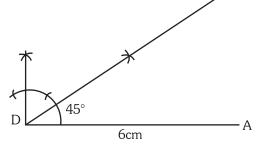
Step I: Draw AD = 6 cm.



Step II : At *D* draw an angle $\angle XDA = 45^{\circ}$ using a compass.



Math-8



10cm

6cm

45°

3cm

5cm

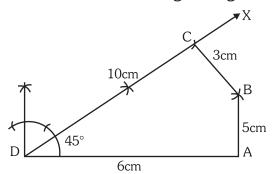
Step III: With D as centre and 10 cm as radius, mark an arc intersecting DX at the point C.

Step IV: With *C* as centre and 3 cm as radius, draw an arc.

Step V: With A as centre and 5 cm as radius, draw another arc intersecting

the previous one at the point *B*.

Step VI: Join BC and AB as shown in the given figure.



The figure ABCD, thus drawn, is the required quadrilateral.

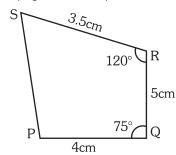
: CONSTRUCTION OF A QUADRILATERAL WHEN ITS THREE SIDES AND TWO INCLUDED ANGLES ARE GIVEN)

Example 4: Construct a quadrilateral PQRS with PQ = 4 cm, QR = 5 cm, RS = 3.5 cm,

 $\angle Q = 75^{\circ}$ and $\angle R = 120^{\circ}$.

Solution: First draw a rough sketch of the figure to

be constructed.

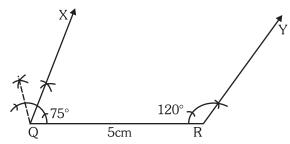


Steps of Construction:

Step I: Draw QR = 5 cm.

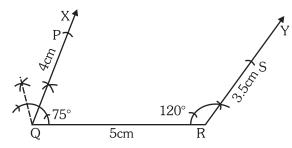
Q 5cm R

Step II: Draw $\angle RQX = 75^{\circ}$ and $\angle QRY = 120^{\circ}$ using a compass.

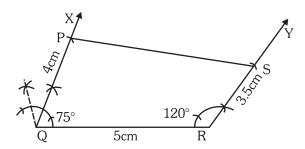


Step III: Using a ruler, locate P on QX, 4 cm away from Q.

Step IV: Using a ruler, locate S on RY, 3.5 cm away from R.



Step V: Join P and S.



The figure *PQRS*, thus drawn, is the required quadrilateral.

CONSTRUCTION OF A QUADRILATERAL WHEN ITS THREE ANGLES AND TWO INCLUDED SIDES ARE GIVEN

Example 5: Construct a quadrilateral TURN in which TU = 5.2 cm, UR = 3.8 cm,

 $\angle T = 100^{\circ}$, $\angle U = 65^{\circ}$ and $\angle R = 130^{\circ}$.

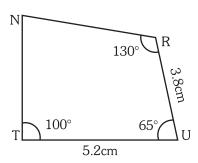
Solution: First draw a rough sketch of the figure

to be constructed.

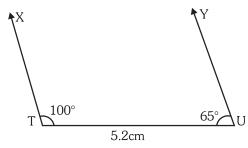
Steps of Construction:

Step I: Draw TU = 5.2 cm.

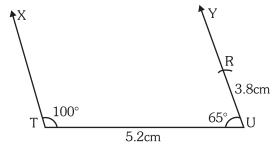
T ______ U



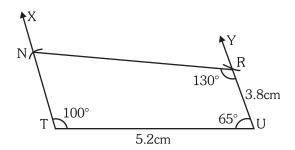
Step II: At point T on line TU, draw $\angle UTX = 100^\circ$. Also at point U on line TU, draw $\angle TUY = 65^\circ$ using a protractor.



Step III: Taking U as centre and UR (= 3.8 cm) as radius, draw an arc on line UY intersecting it at point R.



Step IV: At point R on line UR, draw $\angle URN = 130^\circ$ using a protractor intersecting line TX at N. Join RN.



The figure TURN, thus drawn, is the required quadrilateral.



- 1. Construct a quadrilateral ABCD in which AB = 5.4 cm, BC = 5 cm, CD = 6.2 cm, DA = 4.4 cm and diagonal AC = 7 cm.
- **2.** Construct a quadrilateral SKIP with SK = 5 cm, KI = 6 cm, IP = 5 cm, SI = 5 cm, and KP = 8 cm.
- **3.** Construct a quadrilateral PQRS when PQ = RS = 4 cm, SP = 6.4 cm, QS = 5 cm and PR = 7 cm.
- **4.** Four sides of a quadrilateral are 7 cm, 5 cm, 4.2 cm and 3.6 cm respectively. The angle between the sides 4.2 cm and 3.6 cm is 90°. Construct the quadrilateral.
- **5.** Construct a quadrilateral LIME in which LI = 6 cm, IM = 8 cm, $\angle L = 60^{\circ}$, $\angle I = 120^{\circ}$ and $\angle M = 150^{\circ}$.
- **6.** Construct a quadrilateral *PQRS* in which PQ = QR = RS = 5.5 cm and $\angle Q = \angle R = 90^{\circ}$.
- 7. Construct a quadrilateral *ABCD* in which AB = BC = 6 cm, $\angle B = 135^{\circ}$, $\angle C = 60^{\circ}$ and $\angle D = 55^{\circ}$.
- 8. Construct a quadrilateral MORE in which ME = 4.2 cm, OR = 3.6 cm, RE = 5.4 cm, MR = 7.4 cm and OE = 4 cm.
- **9.** Three sides of a quadrilateral are of lengths 6.5 cm, 6 cm and 4.5 cm respectively and the included angles are of measures 120° and 75°. Construct the quadrilateral.
- **10.** Construct a quadrilateral *PQRS* in which $\angle P = 130^{\circ}$, $\angle Q = 80^{\circ}$, $\angle R = 70^{\circ}$ and PQ = QR = 6.4 cm.

CONSTRUCTION OF SPECIAL TYPES OF QUADRILATERALS

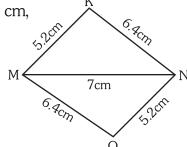
For the construction of the quadrilaterals like parallelogram, rhombus, trapezium, etc. we can use their special features as follows :

- (i) Opposite sides of a parallelogram are equal.
- (ii) Opposite angles of a parallelogram are equal.
- (iii) Diagonals of a parallelogram bisect each other.
- (iv) Diagonals of a rhombus bisect each other at right angles.
- (v) A pair of opposite sides of a trapezium are parallel.
- (vi) Opposite sides of a rhombus are equal.
- (vii) Opposite angles of a rhombus are equal.

Example 6: Construct a parallelogram MONK in which MO = 6.4 cm,

ON = 5.2 cm and diagonal MN = 7 cm.

Solution: First draw a rough sketch of the figure to be constructed.



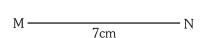
Note: Parallelogram is a quadrilateral in which both pairs of opposite sides are equal.

 \therefore KN = MO = 6.4 cm and KM = ON = 5.2 cm.

Steps of Construction:

Draw a line MN (diagonal) = 7 cm. Step I:

Step II: Taking M as centre and MK (= 5.2 cm) as radius, draw an arc above line MN.



N

Step III: Taking N as centre and NK (= 6.4 cm) as radius, draw another arc intersecting the previous arc at K.



Step IV: Taking M as centre and MO (= 6.4 cm)as radius, draw an arc below line MN.



7cm

7cm

- N





Step V: Taking N as centre and ON (= 5.2 cm) as radius, draw another arc intersecting the arc in step IV at O.



Step VI: Join KM, KN, OM and ON.

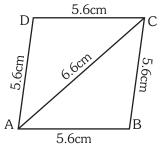
7cm

The figure MONK, thus drawn, is the required parallelogram.

Example 7: Construct a rhombus ABCD with side

AB = 5.6 cm and diagonal AC = 6.6 cm.

Solution: First draw a rough sketch of the figure to be constructed.



Note: Rhombus is a quadrilateral with all sides equal. So, AB = BC = CD = DA = 5.6 cm.

Steps of Construction:

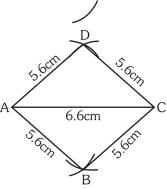
Step I: Draw a line AC (diagonal) = 6.6 cm.

A ______C

Step II: Taking A as centre and AD or AB (= 5.6 cm) as radius, draw two arcs, one above AC and

the other below AC.

Step III: Taking C as centre and CD or CB (= 5.6 cm) as radius, draw two arcs intersecting the arcs in step II at D and B respectively. Join AD, DC, AB and BC.



6.6cm

C

The figure ABCD, thus drawn, is the required rhombus.

Example 8: Construct a parallelogram PURE in which PU = 4.5 cm, UR = 5.5 cm and

 $\angle E = 85^{\circ}$.

Solution: First draw a rough sketch of the figure to be constructed.

We know that opposite angles of a parallelogram are equal.

$$\therefore$$
 $\angle U = \angle E = 85^{\circ}$

Also, *UR* || *PE* and *RE* is the transversal.

$$\therefore$$
 $\angle R + \angle E = 180^{\circ}$

(: Interior angles on the same side of the transversal are supplementary.)

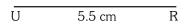
$$\Rightarrow$$
 $\angle R + 85^{\circ} = 180^{\circ}$

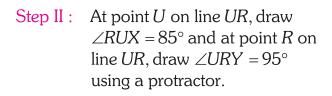
$$\Rightarrow$$
 $\angle R = 180^{\circ} - 85^{\circ}$

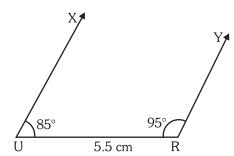
$$\Rightarrow \angle R = 95^{\circ}$$

Steps of Construction:

Step I: Draw a line UR = 5.5 cm.

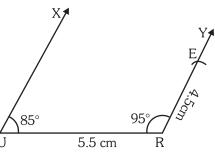


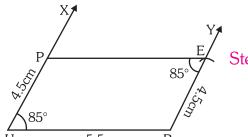




Step III: Taking *R* as centre and RE = 4.5 cmas radius, cut an arc intersecting RY at

point E.





At point E, draw $\angle REP = 85^{\circ}$ using a protractor intersecting UX at point P.

The figure *PURE*, thus drawn, is the required parallelogram.

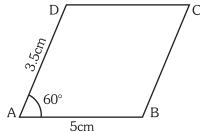
Example 9: Construct a trapezium ABCD in which AB = 5 cm, DC = 4 cm, AD = 3.5 cm

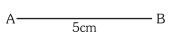
and $\angle A = 60^{\circ}$. It is given that AB is parallel to DC.

Solution: First draw a rough sketch of the figure to be constructed.

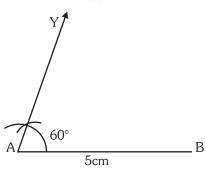
Steps of Construction:

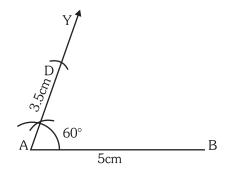
Draw AB = 5 cm. Step I:





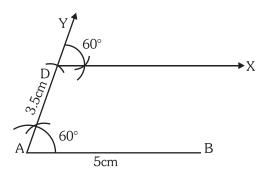
Construct angle $\angle BAY = 60^{\circ}$.



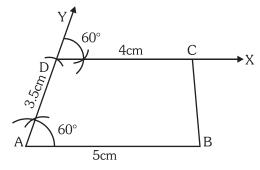


Step III: Locate D on AY at a distance of 3.5 cm from A.

Step IV : Construct $\angle YDX = 60^{\circ}$. This will make $DX \parallel AB$.



Step V: On DX locate a point C such that DC = 4 cm. Join BC.



The figure ABCD, thus drawn, is the required trapezium.



- Construct a parallelogram *BLOW* in which BL = 7 cm, LO = 8 cm and $\angle B = 75^{\circ}$. 1.
- 2. Construct a parallelogram KITE in which KI = 5 cm, IT = 4.6 cm and diagonal KT = 6 cm.
- 3. Construct a rhombus *PQRS* in which two diagonals are of length 8 cm and 6 cm respectively.
- 4. Construct a rhombus ABCD with side AB = 4.7 cm and diagonal AC = 5.8 cm.
- Construct a trapezium ABCD in which $AB \parallel CD$, AB = 6.5 cm, BC = 4.5 cm, CD = 5 cm and **5.** $\angle B = 80^{\circ}$.
- 6. Construct a rectangle ABCD in which AB = 4 cm and AC = 6 cm.

[Note: In a rectangle, all angles are of 90° and opposite sides are equal.]

- 7. Construct a rhombus *MARS* in which MA = 8.4 cm and $\angle M = 75^{\circ}$.
- 8. Construct a square CUBE in which CU = 6.5 cm.

[Note: In a square, all sides are equal and all angles are equal to 90°.]

SUMMARY OF THE CHAPTER

- A quadrilateral is a closed figure having four sides and four angles.
- To construct a quadrilateral, it is sufficient to know five independent parts of the quadrilateral such as:
 - 4 sides and a diagonal
 - 3 sides and both diagonals
 - 2 adjacent sides and three angles
 - 3 sides and two included angles



Review Of The Chapter (Task For Summative Assessment)

- Construct a quadrilateral ABCD in which AB = 3.5 cm, BC = 3.2 cm, CD = 4.1 cm, DA = 4.5 cm 1. and the diagonal AC = 5.2 cm. Measure diagonal BD.
- 2. Construct a quadrilateral PQRS in which PQ = 4 cm, QR = 3 cm, SP = 3.2 cm, PR = 4 cm and QS = 3.8 cm.
- Four sides of a quadrilateral are 7 cm, 5 cm, 4.2 cm and 3.6 cm respectively. The angle between the 3. sides 4.2 cm and 3.6 cm is 90°. Construct the quadrilateral.

- **4.** Construct a quadrilateral *ABCD* in which AB = 5.6 cm, BC = 4.8 cm, $\angle A = 100^{\circ}$, $\angle B = 75^{\circ}$ and $\angle C = 120^{\circ}$.
- **5.** Construct a parallelogram MICE in which MI = 4 cm and MC = 5.2 cm and IE = 5.6 cm.
- **6.** Construct a rhombus whose diagonals are 4 cm and 6 cm respectively.
- **7.** Construct a rectangle *DICE* in which DI = 4.5 cm and diagonal DC = 6 cm.
- 8. Construct a trapezium ABCD in which $AB \parallel CD$, AB = 8.5 cm, BC = 3.6 cm, CD = 3.6 cm and $\angle B = 75^{\circ}$. Multiple Choice Questions (MCQs)

(Task for formative Assessment)

1.	The sum of angles of a quadrilateral is:				
	(a) 90° (b) 180°	(c) 270°	(d)	360°	
2.	To construct a quadrilateral, it is sufficient to h	nave the knowledge o	f at least	independ	dent
	elements.	7			
	(a) 2 (b) 4	」 (c) 5	(d)	6	\bigcup
3.	In a parallelogram, if one angle is 60°, the other	angles are :			
	(a) 60° , 60° , 60° (b) 60° , 90° , 90°	(c) 60°, 120°, 120°	(d)	45°, 135°, 120°	
4.	In a, both pairs of opposite sides are e	equal and all angles ar	e of 90° .		
	(a) square (b) rectangle	(c) parallelogram	(d)	rhombus	
5.	If ABCD is a parallelogram, then $\angle A - \angle C$ is:				
	(a) 0° (b) 90°	(c) 180°	(d)	360°	

Maths Activity

Aim: To check the conditions of the construction of a quadrilateral.

Materials required : Cardboard strip (5 in number), pins (to make hinges), a pair of scissors.

Process:

Step I: Take four cardboard strips of suitable lengths.

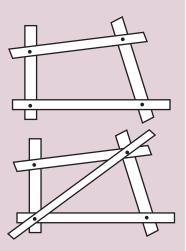
Step II: With the help of pins, hinge the strips at the ends to form a quadrilateral as shown in the figure.

Step III: Try to change the shape of the quadrilateral by pressing at the opposite vertices. Isn't easy! This shows that different quadrilaterals can be made from the same measure of the 4 sides.

Step IV: Add another strip in the form of a diagonal as shown in the figure.

Try to change the shape of the quadrilateral.

Observation: This shows that construction of a unique quadrilateral needs at least 5 building blocks (or dimensions). For example, 4 sides and 1 diagonal, etc.



Mensuration

Sequence of the Chapter

✓ Area Of Rectilinear Figures

✓ Volume

- ✓ Area Of A Rhombus
- ✓ Area Of A Trapezium
- ✓ Area Of A Quadrilateral
- ✓ Area Of A Triangle Using Hero's Formula
- Area Of A Polygon
- ✓ Surface Area

AREA OF RECTILINEAR FIGURES

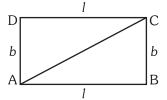
In the earlier classes, we have already learnt how to calculate perimeter and area of various plane figures like rectangle, square, parallelogram, triangle, etc.

In this chapter, we shall learn to find the areas of some other simple closed rectilinear figures such as rhombus, trapezium, quadrilateral, etc. Before finding the areas of these figures, let us revise some formulae and standard units of area and perimeter.

I. Rectangle

Let the length (AB) be l and the breadth (CB) be b, then

- (i) Perimeter (P) = 2(l + b)
- (ii) Area $(A) = l \times b$
- (iii) Diagonal (AC) = $\sqrt{l^2 + b^2}$



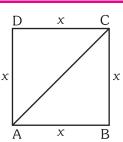
Keep In Mind!

- The part of the plane enclosed within a simple closed rectilinear figure is called the region enclosed by it and the measurement of the region is called its area.
- Perimeter of a simple closed figure is the sum of all the sides.

II. Square

Let the length of the side of the square be x, then

- (i) Perimeter (P) = $4 \times \text{side} = 4x$
- (ii) Area (A) = (side)² = x^2 or $\left(\frac{\text{Perimeter}}{4}\right)^2$ or $\frac{1}{2}$ (Diagonal)²



(iii) Side
$$(x) = \sqrt{Area}$$

(iv) Diagonal (AC) = $\sqrt{2}$ × side = $\sqrt{2}$ x

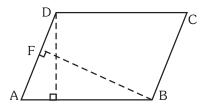
III. Parallelogram

(i) Area $(A) = Base \times Height$

$$= AB \times DE \text{ or } DA \times BF$$

(ii) Perimeter (P) = Sum of all sides

= 2 (Sum of adjacent sides), i. e.,
$$2 (AB + BC)$$



Keep In Mind!

 \blacksquare The standard unit of area is cm² or sq cm and the unit of perimeter is cm or m.

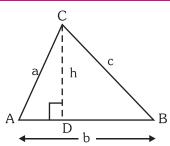
IV. Triangle

In
$$CAB$$
, base $AB = b$, altitude $CD = h$.

(i) Perimeter
$$(P) = Sum \text{ of all sides}$$

$$= a + b + c$$

(ii) Area (A) =
$$\frac{1}{2} \times \text{Base} \times \text{Altitude} = \frac{1}{2} \times b \times h$$



Some standard units of area are given below:

$$100 \text{ mm}^2 = 1 \text{ cm}^2$$

$$100 \text{ cm}^2 = 1 \text{ dm}^2$$

$$100 \text{ dm}^2 = 1 \text{ m}^2$$

$$100 \text{ m}^2 = 1 \text{ are or } 1 \text{ a}$$

$$100 \ a = 1 \ \text{hectare or} \ 1 \ \text{ha}$$

$$100 ha = 1 km^2$$

Example 1: A rectangle of length 30 cm has a perimeter equal to the perimeter of a square of side 20 cm. Which of the two has more area and by how much?

Solution: Length of the rectangle
$$(l) = 30$$
 cm

Let the breadth
$$(b)$$
 be x cm.

Thus, the perimeter of rectangle =
$$2(l + b) = 2(30 + x)$$
 cm

Length of the side of the square
$$= 20 \text{ cm}$$

Perimeter of the square =
$$4 \times side = 4 \times 20 = 80$$
 cm

$$\Rightarrow 2(30+x)=80$$

$$\Rightarrow 30 + x = \frac{80}{2} = 40$$

$$\Rightarrow x = 40 - 30 = 10$$

 \therefore Breadth of the rectangle (b) = 10 cm

Area of rectangle
$$(A_1) = l \times b = 30 \times 10 = 300 \text{ cm}^2$$

Area of square
$$(A_2) = (\text{side})^2 = (20)^2 = 400 \text{ cm}^2$$

$$A_2 > A_1$$

$$A_2 - A_1 = (400 - 300) \text{ cm}^2 = 100 \text{ cm}^2$$

Hence, the area of the square is more than that of the rectangle by $100\ cm^2$.

- **Example 2:** The length and breadth of a rectangular field are in the ratio 5:3. The area of the field is 2,160 sq metre. If a fence is to be made at the rate of $\stackrel{?}{\underset{}{\sim}}$ 250 per metre, how much will it cost?
- **Solution**: Let the length of sides of the field are 5x and 3x. Area of the field = Length × Breadth

$$\Rightarrow$$
 2,160 = 5x × 3x

$$\Rightarrow$$
 2,160 = 15 x^2 or 15 x^2 = 2,160

$$\Rightarrow$$
 $x^2 = \frac{2,160}{15} = 144$

$$\Rightarrow \qquad x = \sqrt{144} = 12$$

.. Length of the field = (5×12) m = 60 m Breadth of the field = (3×12) m = 36 m

To fence the field, it should cover its boundary, i.e., perimeter.

 \therefore Perimeter of the field = 2 (Length + Breadth)

$$= 2 (60 + 36)$$

= 192 m

Cost of fencing the wire per metre = ₹ 250

Cost of fencing 192 m of the field $= ₹ 250 \times 192$ = ₹ 48,000

Hence, the cost of fencing 192 m of the field will be ₹ 48,000.

- **Example 3:** The side of a square exceeds the side of another square by 4 cm and the sum of the areas of the two squares is 400 cm^2 . Find the dimensions of the squares.
- **Solution**: Let the side of square II be x cm.

Then the side of square I = (x + 4) cm

Area of square I $= (x + 4)^2$ cm²

Area of square II $= x^2 \text{ cm}^2$

Sum of the two squares $= 400 \text{ cm}^2$

$$\Rightarrow (x+4)^2 + x^2 = 400$$

$$\Rightarrow x^{2} + 8x + 16 + x^{2} = 400 \qquad [Using (a + b)^{2} = a^{2} + 2ab + b^{2}]$$

$$\Rightarrow$$
 $2x^2 + 8x + 16 - 400 = 0$

$$\Rightarrow 2x^2 + 8x - 384 = 0$$

$$\Rightarrow x^2 + 4x - 192 = 0$$

$$\Rightarrow$$
 $x^2 + 16x - 12x - 192 = 0$ [Factorize by splitting the middle term]

$$\Rightarrow$$
 $x(x+16)-12(x+16)=0$

$$\Rightarrow$$
 $(x - 12)(x + 16) = 0$

$$\Rightarrow$$
 $x-12=0 \text{ or } x+16=0$

$$\Rightarrow$$
 $x = 12 \text{ or } -16$

Since length cannot be negative, we neglect x = -16.

- Length of the side of square II = 12 cm, and Length of the side of square I = (12 + 4) cm = 16 cm
- Example 4: A field is in the form of a triangle. If its area is 2 hectares and the length of its base is 200 m, then find its altitude.
- Since 1 hectare = 10.000 m^2 Solution:

∴ 2 hectares = 20,000 m²
Area of the field =
$$\frac{1}{2}$$
 × Base × Altitude

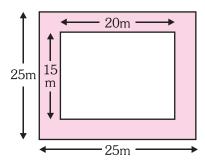
$$\Rightarrow 20,000 = \frac{1}{2} \times 200 \times Altitude$$

$$\Rightarrow \qquad \text{Altitude} = \frac{2 \times 20,000}{200}$$

$$\Rightarrow$$
 Altitude = 200 m

Hence, the altitude of the field is 200 m.

Example 5: Mr. Kashyap has a square plot as shown in the figure. He wants to construct a house in the middle of the plot. A garden is to be developed around the house. Find the total cost of developing the garden, if the cost of developing it is ₹ 75 per square metre.



Area of the square plot = $25 \text{ m} \times 25 \text{ m} = 625 \text{ m}^2$ Solution:

> Area of the plot where house is to be constructed = $20 \text{ m} \times 15 \text{ m} = 300 \text{ m}^2$ Area of the garden = Area of square plot – Area of the plot where house is to be constructed

$$= (625 - 300) \text{ m}^2 = 325 \text{ m}^2$$

Cost of developing 1 m² garden = ₹ 75

Cost of developing 325 m² garden = $\overline{\xi}$ (325 × 75) =₹ 24,375.

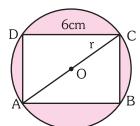


$$(\pi = 3.14, \sqrt{2} = 1.41)$$

Solution: Length of the side of the square = 6 cm

In
$$\triangle CBA$$
, $CA^2 = CB^2 + AB^2$
= $(6)^2 + (6)^2$
= $36 + 36 = 72$

$$\Rightarrow CA = \sqrt{72} = 6\sqrt{2} \text{ cm}$$



(Pythagoras Theorem)

- Diameter of the circle = $6\sqrt{2}$ cm
- Radius of circle $(r) = \frac{6\sqrt{2}}{2} = 3\sqrt{2}$ cm

Area of circle = $\pi r^2 = 3.14 \times (3\sqrt{2})^2 = 3.14 \times 18 = 56.52 \text{ cm}^2$ Area of square = $(Side)^2 = (6)^2 = 36 \text{ cm}^2$

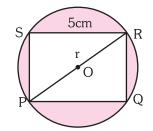
Area of shaded portion = Area of circle - Area of square $= (56.52 - 36) \text{ cm}^2$ $= 20.52 \text{ cm}^2$.

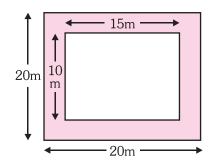


- Find the area of parallelogram whose base is 20 cm and the corresponding height is 5 cm. 1.
- 2. Find the area of the following rectangles, given their lengths and breadths are:
 - (a) 11 cm, 5.6 cm
- (b) 3.85 cm, 4.25 cm
- (c) 18 cm, 20 cm
- 3. Find the area of following squares, given their sides are:
 - (a) 5 cm

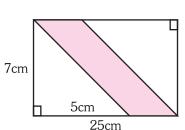
(b) 4.62 cm

- (c) 51 cm
- The length and breadth of a rectangular field are in the ratio 4:3. Its area is 6,912 m². Find the cost 4. of fencing the field at ₹ 2.50 per metre.
- The sides of a triangle are 12 cm, 35 cm and 37 cm. Find its area. **5.**
- Find the altitude of a triangle whose area is 784 cm² and base is 32 cm. 6.
- 7. A square of side 5 cm is inscribed in a circle as shown in the given figure. Find the area of the shaded portion.





- Mrs. Sinha has a square plot as shown in the figure. She wants to construct a house in the middle of the plot. A garden is to be developed around the house. Find the total cost of developing the garden, if the cost of developing it is ₹ 50 per square metre.
- 9. Find the area of shaded portion in the given figure.



The side of a square is 8 cm. Find the length of its diagonal.



We know that diagonals of a rhombus bisect each other at right angles and, therefore, divide the rhombus into four congruent right triangles.

Area of rhombus ABCD $= 4 \times Area$ of right triangle AOB.

$$= 4 \times \frac{1}{2} \times AO \times OB$$

$$= 4 \times \frac{1}{2} \times \frac{1}{2} \times AC \times \frac{1}{2} \times DB$$
$$= 4 \times \frac{1}{4} \times AC \times \frac{1}{2} \times DB = \frac{1}{2} \times AC \times DB$$

Thus, if d_1 and d_2 units are lengths of the diagonals of a rhombus ABCD, then

- (i) Area of the rhombus = $\frac{1}{2} \times d_1 \times d_2$
- (ii) Side of the rhombus = $\frac{1}{2}\sqrt{d_1^2 + d_2^2}$
- (iii) Perimeter of the rhombus = $4 \times \text{side} = 2 \sqrt{{d_1}^2 + {d_2}^2}$

Example 7: Find the area of a rhombus whose diagonals are of lengths 16 cm and 12 cm.

Solution: Area of rhombus
$$=\frac{1}{2} \times d_1 \times d_2$$
 $=\left(\frac{1}{2} \times 16^8 \times 12\right) \text{cm}^2$ $=96 \text{ cm}^2$

Example 8: The area of a rhombus is 80 cm². If one of its diagonals is 10 cm, find the other diagonal.

Solution: Let the length of the other diagonal (d_2) be x cm.

Area of rhombus =
$$\frac{1}{2} \times d_1 \times d_2$$

$$\Rightarrow 80 = \frac{1}{2} \times 10^5 \times x$$

$$\Rightarrow 80 = 5 x$$

$$\Rightarrow 5 x = 80$$

$$\Rightarrow 5 x = 80$$

$$\Rightarrow x = \frac{80}{5} = 16 \text{ cm}$$

Example 9: Find the area of a rhombus, one of whose sides is 18 cm and altitude is 2 cm. If one of its diagonals is 12 cm, find its other diagonal.

Solution: Area of the rhombus = Area of the parallelogram

[Since a rhombus is a parallelogram.]

= Base
$$\times$$
 Height
= (18×2) cm² = 36 cm²

Also, area of the rhombus = $\frac{1}{2} \times d_1 \times d_2$

One of the diagonals is 12 cm and let the other be x cm.

$$\Rightarrow \frac{1}{2} \times 12 \times x = 36$$

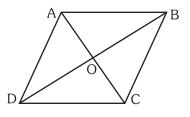
$$\Rightarrow x = \frac{{}^{3}36 \times 2}{12} = 6 \text{ cm}$$

Hence, the other diagonal of the rhombus is 6 cm.

Example 10: If the area of a rhombus is 48 cm^2 and one of its diagonals is 6 cm, find its altitude.

Solution: Let ABCD be a rhombus whose diagonal AC = 6 cm.

Now, area of rhombus ABCD given = 48 cm^2 $\Rightarrow \frac{1}{2} \times AC \times DB = 48$ $\Rightarrow \frac{1}{2} \times 6^3 \times DB = 48$ $\Rightarrow DB = \frac{48}{3} = 16 \text{ cm}$



Since diagonals of a rhombus bisect each other at right angle, we have

OA =
$$\frac{1}{2}AC = \frac{1}{2} \times 6^{3} \text{ cm} = 3 \text{ cm}$$
, and
OB = $\frac{1}{2}DB = \frac{1}{2} \times 16^{8} \text{ cm} = 8 \text{ cm}$

Also, \triangle *AOB* is a right triangle, where \angle *O* = 90°.

$$AB = \sqrt{OA^2 + OB^2} = \sqrt{(3)^2 + (8)^2} \text{ cm} = \sqrt{9 + 64} \text{ cm} = \sqrt{73} \text{ cm}$$

Now, AB can be taken as a base of the rhombus ABCD.

Then, area of rhombus $ABCD = 48 \text{ cm}^2$

$$\Rightarrow$$
 Base × Altitude = 48 cm²

$$\Rightarrow$$
 AB × Altitude = 48 cm²

$$\Rightarrow$$
 $\sqrt{73}$ cm × Altitude = 48 cm²

$$\Rightarrow \qquad \text{Altitude} = \frac{48}{\sqrt{73}} \text{ cm}$$

Hence, altitude of the rhombus is $\frac{48}{\sqrt{73}}$ cm.

AREA OF A TRAPEZIUM

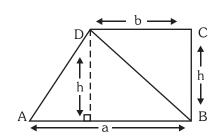
A trapezium is a quadrilateral in which one pair of opposite sides are parallel. Each of the two parallel sides is called the 'base' of the trapezium. The distance between the two parallel sides of the trapezium is the 'altitude' or 'height' of the trapezium.

In the given figure, we have

Area of
$$\triangle$$
 $ABD = \frac{1}{2} \times \text{Base} \times \text{Height}$

$$= \frac{1}{2} \times AB \times h$$

$$= \frac{1}{2} \times a \times h$$



Area of
$$\triangle$$
 $BCD = \frac{1}{2} \times Base \times Height$
= $\frac{1}{2} \times CD \times h$
= $\frac{1}{2} \times b \times h$

∴ Area of trapezium
$$ABCD$$
 = Area of \triangle ABD + Area of \triangle BCD = $\left(\frac{1}{2} \times a \times h\right) + \left(\frac{1}{2} \times b \times h\right) = \frac{1}{2} \times h \times (a + b)$

Thus, the area of a trapezium = $\frac{1}{2}$ × (Sum of lengths of parallel sides) × Altitude

Example 11: The area of a trapezium is 440 cm². If the parallel sides are 7 cm and 9 cm long, find the distance between them.

Solution: Area of trapezium = 440 cm^2 Length of parallel sides = 7 cm and 9 cm

Let the distance between the parallel sides be $x\ \mathrm{cm}$.

Area of trapezium $= \frac{1}{1} \times (Sum \text{ of langths of parallel sides}) \times (Distance)$

$$= \frac{1}{2} \times (\text{Sum of lengths of parallel sides}) \times (\text{Distance between them})$$

$$440 = \frac{1}{2} \times (7 + 9) \times x$$

$$\Rightarrow \frac{1}{2} \times 16^{8} \times x = 440$$

$$\Rightarrow 8 \times x = 440$$

$$\Rightarrow x = \frac{440}{8} = 55 \text{ cm}$$

Hence, the distance between the parallel sides is 55 cm.

Example 12: The ratio of the length of the parallel sides of a trapezium is 3:2. The distance between them is 10 cm. If the area of the trapezium is 600 cm², find the lengths of the parallel sides.

Solution: Let the length of the two parallel sides be 3x and 2x respectively. Distance between them = 10 cm

Area of trapezium = 600 cm^2

Area of trapezium

 $=\frac{1}{2}\times$ (Sum of lengths of parallel sides)× (Distance between them)

$$\Rightarrow \qquad 600 = \frac{1}{2} \times (3x + 2x) \times 10$$

$$\Rightarrow 600 = \frac{1}{2} \times 5x \times 10^{5}$$

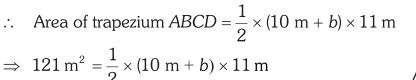
$$\Rightarrow 5x \times 5 = 600 \qquad \Rightarrow x = \frac{600}{5 \times 5} = 24$$

Hence, the lengths of the two parallel sides are 3 \times 24, i.e., 72 cm and 2 \times 24, i.e., 48 cm.

- **Example 13**: The area of a trapezium of height 11 m is 121 sq m. If one of the parallel sides is 10 m, what is the length of the other side?
- **Solution**: We are given that area of the trapezium = 121 sq m.

Height or altitude = 11 m One parallel side = 10 m Let the other parallel side be b m.

Then, area of the trapezium = $\frac{1}{2}$ × (Sum of lengths of parallel sides) × Altitude



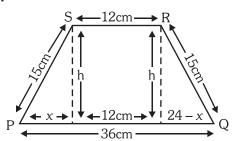
$$\Rightarrow \frac{^{11}121 \, \text{m}^8 \times 2}{11 \, \text{m}} = 10 \, \text{m} + b$$

$$\Rightarrow$$
 22 m = 10 m + b \Rightarrow b = 22 m - 10 m = 12 m

Hence, the length of the other side of the trapezium is 12 m.



- 1. The area of a rhombus is 256 cm². If one of the diagonals measures 16 cm, find the length of the other diagonal.
- 2. Find the area of rhombus whose diagonals are 18 cm and 21 cm.
- **3.** Find the area of the rhombus if the length of each side be 7 cm and the corresponding altitude be 8 cm.
- **4.** The area of a trapezium is 400 cm². If its height is 40 cm and the length of one of its base is 10 cm, find the length of its other base.
- 5. Find the area of a rhombus whose perimeter is 60 cm and one diagonal is of length 18 cm.
- 6. The parallel sides of a trapezium are $12 \, \text{m}$ and $6 \, \text{m}$ respectively and the distance between them is $20 \, \text{m}$. What is the area of the trapezium?
- 7. The area of a trapezium of height 10 m is 250 sq m. If one of the parallel sides is 14 m, what is the length of the other side?
- 8. The area of a trapezium is 138 m². The distance and the difference between the lengths of the parallel sides are 12 m and 7 m respectively. Find the lengths of the parallel sides.
- **9.** The ratio of the length of the parallel sides of a trapezium is 6 : 2. The distance between them is 32 cm. If the area of the trapezium is 384 cm², find the lengths of the parallel sides.
- **10.** Find the area of a trapezium whose parallel sides are 36 cm and 12 cm, and the non-parallel sides are 15 cm each.



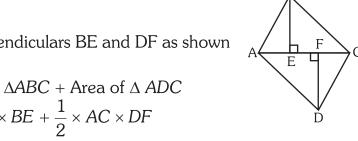
/ 121 m²



AREA OF A QUADRILATERAL

Let us consider a quadrilateral ABCD.

Draw its diagonals AC and draw the perpendiculars BE and DF as shown in the figure.



Area of quadrilateral ABCD

= Area of
$$\triangle ABC$$
 + Area of $\triangle ADC$
= $\frac{1}{2} \times AC \times BE + \frac{1}{2} \times AC \times DF$
= $\frac{1}{2} \times AC \times (BE + DF)$
= $\frac{1}{2} \times \text{diagonal} \times \text{sum of perpendicular distance from the opposite vertices to this diagonal.}$

The perpendicular distance from the other vertices are called **offsets** of these vertices to the diagonal.

$$\therefore$$
 Area of a quadrilateral = $\frac{1}{2} \times \text{diagonal} \times \text{sum of offsets}$

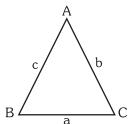
Example 14: Find the area of a quadrilateral in which the length of one of the diagonals is 42 cm and perpendiculars (i.e., offsets) drawn to it from the opposite vertices are 9.5 and 1.5 respectively.

Solution: Area of a quadrilateral = $\frac{1}{2}$ × diagonal × sum of the offsets

$$\therefore \text{ Area of the given quadrilateral } = \frac{1}{2} \times 42^{21} \times (9.5 + 1.5)$$
$$= 21 \times 11.0$$
$$= 231 \text{ cm}^2$$

AREA OF A TRIANGLE USING HERO'S FORMULA

Greek mathematician *Hero* deduced a formula for finding out the area of a triangle in terms of its sides. The formula is given below :



Area of triangle
$$ABC = \sqrt{s(s-a)(s-b)(s-c)}$$

Where a, b, c are the lengths of the sides of the triangle and s (the semi-perimeter) = $\frac{a+b+c}{2}$.

Keep In Mind!

All the three sides of an equilateral triangle are equal. Let each of them be x, i.e., a = b = c = x.

$$\therefore \quad \text{Semi-perimeter (s)} = \frac{x + x + x}{2} = \frac{3}{2}x$$

Thus, area (A) of an equilateral triangle is given by:

$$A = \sqrt{\frac{3x}{2} \left(\frac{3}{2}x - x\right) \left(\frac{3}{2}x - x\right) \left(\frac{3}{2}x - x\right)} = \sqrt{\frac{3x}{2} \left(\frac{x}{2}\right) \left(\frac{x}{2}\right) \left(\frac{x}{2}\right)}$$

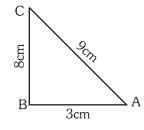
$$=\sqrt{\frac{3x^4}{16}} = \sqrt{3\left(\frac{x^2}{4}\right)^2} = \frac{\sqrt{3}}{4}x^2 = \frac{\sqrt{3}}{4}\left(\text{side}\right)^2$$

Example 15: The lengths of the sides of a triangle are 9 cm, 3 cm and 8 cm. Find its area.

Solution: Here,
$$a = 8$$
 cm,

$$b = 3$$
 cm and $c = 9$ cm

$$\therefore s = \frac{a+b+c}{2} = \frac{8+3+9}{2} \text{ cm}$$
$$= \frac{20}{9}^{10} \text{ cm} = 10 \text{ cm}$$



Therefore, area of the triangle,

$$A = \sqrt{s (s - a) (s - b) (s - c)} = \sqrt{10 (10 - 8) (10 - 3) (10 - 9)}$$
$$= \sqrt{10 \times 2 \times 7 \times 1} = \sqrt{140}$$
$$= 2 \sqrt{35} \text{ cm}^{2}.$$

Example 16: Find the area of an isosceles triangle whose equal sides measure 5 cm each and the third side is 4 cm.

$$AB = AC$$
, i.e., $c = b$.

It is given that,
$$a = 4$$
 cm, $b = 5$ cm, $c = 5$ cm

$$s = \frac{4+5+5}{2}$$
 cm $= \frac{14}{2}$ cm $= 7$ cm

Now, area of the triangle =
$$\sqrt{s(s-a)(s-b)(s-c)}$$

= $\sqrt{7(7-4)(7-5)(7-5)} = \sqrt{7 \times 3 \times 2 \times 2}$
= $\sqrt{21 \times 2^2}$
= $2\sqrt{21}$ cm².

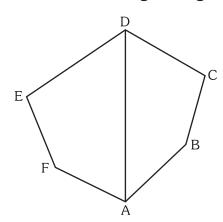
AREA OF A POLYGON

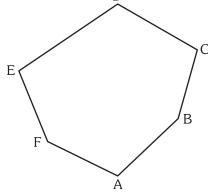
Observe the shapes and sizes of some plots and fields in your surroundings. You will notice that they are not always in the form of squares, rectangles or triangles. Some of them are of the shape of irregular polygons. Then, how can we calculate the area of such polygons? Let us try.

We can find an irregular polygon into triangles or quadrilaterals, or a combination of the two, and then find the area by applying the formulae already known.

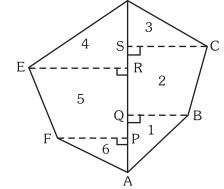
Suppose we have to calculate the area of a field ABCDEF as given alongside.

First draw the longest diagonal AD in the polygon ABCDEF.





Then, draw perpendiculars from each of its remaining vertices on *AD*. The polygon is now divided into right triangles and trapeziums as shown below.



12

10

10

12

Now, Area of polygon $ABCDEF = Area of \Delta AQB + Area$

of trapezium QBCS + Area of ΔDSC + Area of ΔDRE + Area of trapezium EFPR + Area of ΔAPF

i.e., Area of the polygon = Area of regions (1 + 2 + 3 + 4 + 5 + 6)

Example 17: Find the area of the polygon *ABCDE* whose measurements in metre are given in the figure.

Solution: Area of polygon ABCDE

= Area of $\triangle ABE$ + Area of $\triangle BCE$ + Area of $\triangle CDE$

In $\triangle ABE$, s (semi-perimeter) = $\frac{8+8+12}{2}$ m = $\frac{28}{2}$ m = 14 m

∴ Area of
$$\triangle ABE = \sqrt{14 (14 - 8) (14 - 8) (14 - 12)} \text{ m}^2$$

$$= \sqrt{14 \times 6 \times 6 \times 2} \text{ m}^2$$

$$= \sqrt{2 \times 7 \times 6 \times 6 \times 2} \text{ m}^2$$

$$= \sqrt{7 \times 2^2 \times 6^2} \text{ m}^2$$

$$= 2 \times 6 \times \sqrt{7} \text{ m}^2$$

$$= 12 \times 2.645 \text{ m}^2 = 31.74 \text{ m}^2$$

In
$$\triangle BCE$$
, s (semi-perimeter) = $\frac{12 + 12 + 8}{2}$ m = $\frac{32}{2}$ m = 16 m

∴ Area of
$$\triangle BCE = \sqrt{16 (16 - 12) (16 - 12) (16 - 8)} \text{ m}^2$$

$$= \sqrt{16 \times 4 \times 4 \times 8 \text{ m}^2}$$

$$= \sqrt{4^2 \times 4^2 \times 2^2 \times 2 \text{ m}^2}$$

$$= 4 \times 4 \times 2 \times \sqrt{2} \text{ m}^2$$

$$= 32 \times 1.414 \text{ m}^2 = 45.25 \text{ m}^2$$

In
$$\triangle CDE$$
, s (semi-perimeter) = $\frac{10 + 10 + 12}{2}$ m = $\frac{32}{2}$ m = 16 m

.. Area of
$$\triangle CDE = \sqrt{16 \ (16-10) \ (16-10) \ (16-12)} \ m^2$$

$$= \sqrt{16 \times 6 \times 6 \times 4} \ m^2$$

$$= \sqrt{4^2 \times 6^2 \times 2^2} \ m^2$$

$$= 4 \times 6 \times 2 \ m^2$$

$$= 48 \ m^2.$$

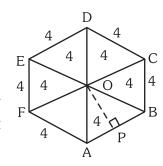
Thus, area of the polygon $ABCDE = 31.74 \text{ m}^2 + 45.25 \text{ m}^2 + 48 \text{ m}^2$ $= 124.99 \text{ m}^2$.

Example 18: Find the area of a regular hexagon with side 4 units.

Solution: As shown in the figure, the hexagon can be divided into six equilateral triangles with side 4 units.

Thus, area of the hexagon = $6 \times$ area of $\triangle AOB$,

To find the area of $\triangle AOB$, we draw the altitude (or height) *OP* from point O to the base *AB*. We know that the altitude of an equilateral divides the base into two. Thus, P is the midpoint of AB.



Thus,
$$PB = \frac{4}{2} = 2$$
 units and $OB = 4$ units

Now,
$$OP^2 + PB^2 = OB^2$$
 [By Pythagoras theorem]

Thus, the height
$$OP = \sqrt{OB^2 - PB^2} = \sqrt{(4^2) - (2^2)}$$

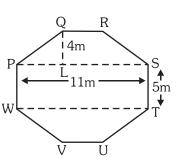
$$= \sqrt{16 - 4} = \sqrt{12} = 2\sqrt{3} \text{ units}$$

$$=\sqrt{16-4}=\sqrt{12}=2\sqrt{3} \text{ units}$$
 The area of $\triangle AOB=\frac{1}{2}\times OP\times AB=\frac{1}{2}\times 2\sqrt{3}\times 4=4\sqrt{3}$ units

Thus, the area of hexagon = $6 \times (Area \text{ of } \triangle AOB)$

$$=6 \times 4\sqrt{3} = 24\sqrt{3}$$
 sq units.

- **Example 19**: Top surface of a raised platform is in the shape of a regular octagon. Find the area of the octagonal surface.
- Solution: Area of octagon PQRSTUVW = (Area of trapezium)PQRS) + (Area of rectangle PSTW) + (Area of trapezium WTUV)

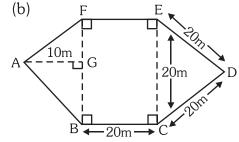


= 2 (Area of trapezium
$$PQRS$$
) + (Area of rectangle $PSTW$)
= $2\left\{\frac{1}{2} \times (QR + PS) \times QL\right\} + (WT \times ST)$
= $2\left\{\frac{1}{2} \times (5 + 11) \times 4\right\} + (11 \times 5)$
= $2\left(\frac{1}{2} \times 16 \times 4\right) + (11 \times 5)$
= $64 + 55 = 119 \text{ m}^2$.

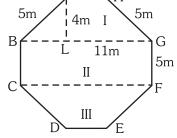
Exercise: 14 (C)

- 1. The diagonal of a quadrilateral is 30 cm and the perpendiculars drawn to the diagonal from the opposite vertices are of lengths 9.2 cm and 5.5 cm. Find the area of the quadrilateral.
- 2. Find the length of a diagonal of a quadrilateral whose area is 180 m² and the lengths of the perpendiculars to the diagonal from opposite vertices are 8 m and 4 m respectively.
- 3. Find the area of the following polygons:

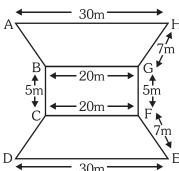
(a) F E 5cm C D 4cm 4cm



4. Top surface of a raised platform is in the shape of a regular octagon as shown in the figure. Find the area of the octagonal surface.



- 5. The lengths of the sides of a triangle are 13 cm, 14 cm and 15 cm. Find the length of the altitude corresponding to the longest side.
- **6.** Find the area of the following triangles whose sides are:
 - (a) 58 cm, 162 cm and 112 cm
 - (b) 40 cm, 41 cm and 11 cm
- **7.** Find the area enclosed by the following polygon. (Use $\sqrt{3} = 1.73$)



SURFACE AREA

The surface area of every solid is the sum of the areas of all the faces or surfaces that enclose the solid. The faces include the tops and the bottoms called 'bases', and the remaining surfaces are called 'lateral faces' or 'surfaces'.

In this section, we will learn how to find the surface area of cubes, cuboids and cylinders.

Surface Area Of A Cube

The six faces of a cube are squares of the same size, i.e., l = b = h = a (edges of cube).

 \therefore Surface area of a cube = 2 ($a \times a + a \times a + a \times a$) sq units

=
$$2 (a^2 + a^2 + a^2)$$
 sq units
= $2 (3a^2)$ sq units

 $=6a^2$ sq units

 $=6 \text{ (side)}^2 \text{ sq units}$

i.e., Surface area of a cube = $6 \times (\text{side})^2$

Length of the diagonal of a cube $=\sqrt{3} \ a$ units $=\sqrt{3}$ (Edge) units

Lateral surface area of a cube $= 2 \times a (a + a)$ sq units

=2a(2a) sq units

 $=4a^2$ sq units =4 (Edge)² sq units

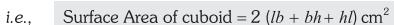
Surface Area Of A Cuboid

A cuboid consists of 6 rectangular faces. So, its surface area is equal to the sum of the areas of its 6 rectangular faces.

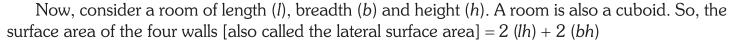
Let there be a cuboid of length l cm, breadth b cm and height h cm.

- (i) Area of face ABFG = Area of face $DCEH = 1 \times b$ cm²
- (ii) Area of face $ADHG = Area of face <math>BCEF = b \times h cm^2$
- (iii) Area of face ABCD = Area of face $HEFG = l \times h$ cm² Total Surface Area of the cuboid

$$= 2 (lb) + 2 (bh) + 2 (hl)$$



Length of the diagonal (AE) =
$$\sqrt{l^2 + b^2 + h^2}$$
 cm



$$\therefore$$
 Lateral Surface Area = $2h (l + b)$ sq units = P erimeter \times Height sq units

Example 20: Find the side of a cube whose surface area is 1536 cm².

Solution: Let side of the cube be x cm.

 \therefore Surface area of the cube = $6x^2$

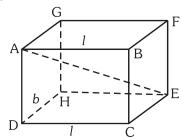
$$\Rightarrow 1536 = 6x^2$$

$$\Rightarrow$$
 $6x^2 = 1536$

$$\Rightarrow x^2 = \frac{1536}{6} = 256$$

$$\Rightarrow$$
 $x = \sqrt{256} = 16$

Thus, side of the cube is 16 cm.



Example 21: Find the total surface area, lateral surface area and the length of the diagonal

of a cuboid of dimensions $0.8 \ dm \times 5 \ cm \times 10 \ cm$.

Solution: Length (*I*) = $0.8 dm = (0.8 \times 10) cm = 8 cm$,

Breadth (b) = 5 cm, Height (h) = 10 cm

Total surface area
$$=2(lb + bh + hl)$$

$$= 2 (8 \times 5 + 5 \times 10 + 10 \times 8)$$

$$= 2 (40 + 50 + 80)$$

$$= 2 \times 170 = 340 \text{ cm}^2$$

Lateral surface area = 2 h (l + b)

$$= 2 \times 10 \times (8+5)$$

$$=2\times10\times13$$

$$= 260 \text{ cm}^2$$

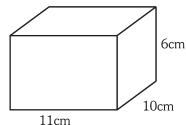
Length of diagonal =
$$\sqrt{l^2 + b^2 + h^2} = \sqrt{(8)^2 + (5)^2 + (10)^2}$$

= $\sqrt{64 + 25 + 100} = \sqrt{189}$

$$= 3\sqrt{21} \text{ cm or } 13.75 \text{ cm (approx.)}$$

Example 22: A cuboid has the dimensions as shown in the adjoining figure. Find its:

- (a) total surface area
- (b) area of four walls



Solution: Dimensions of the cuboid are given by :

length (l) = 11cm, breadth (b) = 10 cm, height (h) = 6 cm

(a) Total surface area of the cuboid = 2(lb + bh + hl)

$$= 2 (11 \times 10 + 10 \times 6 + 6 \times 11)$$

$$= 2(110 + 60 + 66)$$

$$= 2 \times 236$$

$$=472 \text{ cm}^2$$
.

(b) Area of four walls of the cuboid = $2(l + b) \times h$

$$= 2 (11 + 10) \times 6$$

$$=2\times21\times6$$

$$= 252 \text{ cm}^2.$$

- **Example 23**: A suitcase with measures 80 cm × 48 cm × 24 cm is to be covered with a tarpaulin cloth. How many metres of tarpaulin of width 96 cm is required to cover 100 such suitcases?
- **Solution**: Total surface area of the suitcase

$$= 2 (lb + bh + hl)$$

$$= 2 (80 \times 48 + 48 \times 24 + 24 \times 80)$$

$$= 2 (3840 + 1152 + 1920)$$

$$= 2 \times 6912$$

$$= 13824 \text{ cm}^2$$

Total surface area of the suitcase

= Area of the tarpaulin required to cover 1 suitcase

Let x cm of tarpaulin of width 96 cm is required to cover 1 suitcase.

$$\Rightarrow$$
 13824 = $x \times 96$

$$\Rightarrow x = \frac{13824}{96} = 144 \text{ cm}$$

Length of tarpaulin required for 100 such suitcases

$$= 144 \times 100 \text{ cm}$$

$$= \left(\frac{144 \times 100}{100}\right) \text{m} = 144 \text{ m}.$$

Example 24: The total surface area of a cuboid is 40 m² and its lateral surface area is 26 m². Find the area of the floor.

Solution: Surface area of a cuboid = 2 (lb + bh + hl)

$$=2lb+2(l+b)h$$

Surface area = $2 \times$ area of floor + lateral surface area

$$40 = 2 \times area of floor + 26$$

$$\Rightarrow$$
 2 × area of floor = (40 – 26) m² = 14 m²

$$\therefore \text{ Area of floor} = \frac{14^7}{2} \text{ m}^2 = 7 \text{ m}^2$$

Hence, the area of the floor is 7 m².

Example 25: The dimensions of an encyclopedia are 24 cm × 12 cm × 6 cm. It is to be covered with a plastic sheet. If each encyclopedia requires 170 cm² of extra sheet for folding, how much plastic sheet is required to wrap 50 such encyclopedias?

Solution: Length of encyclopedia (l) = 24 cm

Breadth of encyclopedia (b) = 12 cm

Height of encyclopedia (h) = 6 cm

The encyclopedia is to be covered by a plastic sheet only on three faces *i.e.*, top, bottom and spine (the bound side).

Area of top = $l \times b = 24 \text{ cm} \times 12 \text{ cm} = 288 \text{ cm}^2$

Area of bottom = $l \times b = 24 \text{ cm} \times 12 \text{ cm} = 288 \text{ cm}^2$

Area of spine = $l \times h = 24 \text{ cm} \times 6 \text{ cm} = 144 \text{ cm}^2$

Area of extra sheet required for folding = 170 cm^2

Area of plastic sheet required for 1 encyclopedia

$$= (288 + 288 + 144 + 170) \text{ cm}^2$$

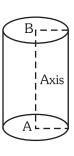
= 890 cm^2

: Area of plastic sheet required for 50 such encyclopedias

$$= (50 \times 890) \text{ cm}^2 = 44.500 \text{ cm}^2$$

Surface Area Of A Cylinder

Let's consider objects like a gas cylinder, coke cans, etc. These solids have a curved lateral surface with congruent circular ends. Such solids are **right circular cylinders**. A right circular cylinder has two plane ends, also called the 'base' of the cylinder. The two bases are circular in shape and parallel to each other. The line segment joining the centres of the two bases is called the 'axis' of the cylinder. The axis of the cylinder is perpendicular to the circular ends.



Lateral surface area or curved surface area

- = Area of the rectangle (assuming the cylinder is cut vertically and rolled out)
- = Area of the rectangle with length ' $2\pi r$ ' and breadth 'h'
- $=2\pi rh$ sq units

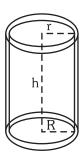
Base surface area $= \pi r^2$ sq units

Total surface area = Curved surface area + Area of the two bases

$$=(2\pi rh+2\pi r^2)$$
 sq units

$$=2\pi r (h+r)$$
 sq units

Surface area of a hollow cylinder: Objects like piece of rubber tube, piece of iron pipe, etc. are hollow cylinders. A solid bounded by two co-axial cylinders of same height and different radii is called a **hollow cylinder**. In the adjoining figure, R and r are the external and internal radii respectively and h is the height of the hollow cylinder.



- (i) Surface area of each base = $\pi (R^2 r^2)$ sq units
- (ii) Lateral surface area or curved surface area
 - = External surface area + Internal surface area

$$=(2\pi Rh+2\pi rh)$$
 sq units

$$=2\pi h (R+r)$$
 sq units

(iii) Total surface area = Curved surface area + Surface area of two bases

$$= \{2\pi h (R + r) + 2\pi (R^2 - r^2)\}$$
 sq units

$$= \{2\pi h (R+r) + 2\pi (R+r) (R-r)\}$$
 sq units

$$= \{2\pi \; (R+r) \; (h+R-r)\} \; \text{sq units}$$

Example 26: Find the radius of the base of a right circular cylinder whose curved surface

area is 352 sq cm and height is 16 cm.

Solution: Let r be the radius of the base of the cylinder. We are given that h=16 cm, and curved surface area = $2\pi rh=352$ sq cm

$$\Rightarrow 2 \times \frac{22}{7} \times r \times 16 = 352$$

$$\Rightarrow r = \frac{352 \times 7}{2 \times 22 \times 16} = 3.5$$

Hence, the radius of the base is $3.5\ \text{cm}$.

Example 27: Find the area levelled by a cylindrical roller of diameter 60 cm and length 3.2 m in 120 revolutions.

Solution: Diameter = 60 cm

:. Radius
$$(r) = \frac{60}{2} = 30 \text{ cm} = \frac{30}{100} = 0.3 \text{ m}$$
 $(:: 1 \text{ cm} = \frac{1}{100} \text{ m})$

Height
$$(h) = 3.2 \text{ m}$$

Area levelled in 1 revolution =
$$2\pi rh = 2 \times \frac{22}{7} \times 0.3 \times 3.2 \text{ m}^2$$

$$=6.03 \text{ m}^2 \text{ (approx.)}$$

Area levelled in 120 revolutions = $120 \times 6.03 = 723.6 \text{ m}^2$.

Example 28: A 20 m long iron pipe has exterior diameter equal to 25 cm. If the thickness of the pipe is 1 cm, find the whole surface area of the pipe.

External radius, $R = \frac{25}{2} = 12.5$ cm Solution:

Internal radius, r = outer radius - thickness = (12.5 - 1) cm = 11.5 cm

Height of the cylinder = 20 cm

Total surface area of the pipe = External curved surface area + Internal curved surface area +2 (Area of the base)

$$= 2\pi Rh + 2\pi rh + 2\pi (R^2 - r^2)$$

$$= 2\pi (R + r) h + 2\pi (R^2 - r^2)$$

$$= 2 \times \frac{22}{7} (12.5 + 11.5) 20 + 2 \times \frac{22}{7} \quad \{(12.5)^2 - (11.5)^2\}$$

$$= 2 \times \frac{22}{7} \times 24 \times 20 + 2 \times \frac{22}{7} \times 24$$

$$= 2 \times \frac{22}{7} \times 24 \times (20 + 1)$$

$$= 2 \times \frac{22}{7} \times 24 \times 21 = 3,168 \text{ cm}^2$$

Hence, whole surface area of the pipe is 3,168 cm².

Example 29: Two cylinders have their radii in the ratio 4:5 and their heights in the ratio 5: 6. Find the ratio of their curved surface area.

Solution: Let the radii of the two cylinders be 4r and 5r, and heights be 5h and 6h respectively.

Let
$$s_1$$
 and s_2 be the curved surface area of the two cylinders.

$$\therefore \frac{s_1}{s_2} = \frac{2\pi r_1 h_1}{2\pi r_2 h_2} = \frac{r_1 h_1}{r_2 h_2} = \frac{4r \times 5h}{5r \times 6h} = \frac{4}{6} = \frac{2}{3}$$

$$\therefore \quad s_1: s_2 = 2:3$$

Hence, the ratio of the curved surface area = 2:3.



Find the surface area of a cuboid whose: 1.

(a) length = 8 cm, breadth = 11 cm, height = 17 cm

- (b) length = 4.2 m, breadth = 3 m, height = 150 cm
- **2.** Find the surface area of a cube of side 4.2 m.
- **3.** A swimming pool is 20 m long, 15 m broad and 4 m deep. Find the cost of cementing its floor and the walls at the rate of ₹ 15 per square metre.
- **4.** The dimensions of a cube are doubled. By how many times will its surface area increase?
- 5. A room has a length of 8 m and breadth of 6 m. If the cost of painting its four walls at the rate of 80 per m^2 is $\stackrel{?}{\underset{?}{|}}$ 11,200, find the height of the room.
- 6. A cuboid has a total surface area of 149 m² and its lateral surface area is 135 m². Find the area of its base.
- 7. A cubical cardboard box of side 8 cm is made. Find the area of the coloured paper required to cover the box. If the cost of the paper measuring 6 cm × 4 cm is ₹ 96, find the cost of paper required to cover the box.
- 8. Find the curved surface area of a right circular cylinder of base radius 3 cm and height 5 cm. (Take $\pi = 3.14$)
- **9.** The inner diameter of a circular well is 3.5 m and it is 12 m deep. Find the cost of plastering the inner surface at the rate of ₹ 4 per square metre.
- **10.** The ratio of the curved surface area and total surface area of a right circular cylinder is 3 : 5. Find the ratio of its height to its radius.
- 11. The diameter of a 120 m long roller is 84 cm. If it takes 1000 complete revolutions to level a playground having breadth of 32 m, find the length of the playground.
- **12.** An iron pipe 20 cm long has external diameter equal to 25 cm. If the thickness of the pipe is 1 cm, find the whole surface of the pipe.



The magnitude of the space occupied by a solid figure is called its volume. All solid figures are 3-dimensional *i.e.*, they have length, breadth and height as dimensions. These are measured in metres, centimetres, millimetres, etc. Therefore, the volume of a solid figure is expressed in cubic metres (m³), cubic centimetres (cm³), cubic millimetres (mm³), etc. One cubic centimetre (1 cm³) means the space occupied by a cube of side 1 cm. One cubic metre (1 m³) means the amount of space occupied by a cube of side 1 m.One cubic millimetre means the amount of space occupied by a cube of side 1 mm.

```
Thus, 1 \text{ m}^3 = 1 \text{ m} \times 1 \text{ m} \times 1 \text{ m}

1 \text{ cm}^3 = 1 \text{ cm} \times 1 \text{ cm} \times 1 \text{ cm}

1 \text{ mm}^3 = 1 \text{ mm} \times 1 \text{ mm} \times 1 \text{ mm}
```

Volume Of A Cube

Volume of a cube = $(side)^3$

Volume Of A Cuboid

Volume of a cuboid = (Area of the base) \times (height) = length (l) \times breadth (b) \times height (h) = $l \times b \times h$

Example 30: How many bricks will be required for a wall which is 8 m long, 6 m high and 22.5 cm thick, if each brick measures 25 cm $\times 11.25$ cm $\times 6$ cm?



Solution: Length of the wall = (8×100) cm = 800 cm

Breadth of the wall = 22.5 cm

Height of the wall $= (6 \times 100) \text{ cm} = 600 \text{ cm}$ Volume of the wall $= (800 \times 22.5 \times 600) \text{ cm}^3$ Volume of 1 brick $= (25 \times 11.25 \times 6) \text{ cm}^3$

Number of bricks required

 $= \frac{\text{Volume of the wall}}{\text{Volume of one brick}}$ $= \frac{800 \times 22.5 \times 600}{25 \times 11.25 \times 6} = 6400$

Example 31: Find the volume of a cube whose side is 6 cm.

Solution: Side of cube = 6 cm

Volume of cube = $(side)^3 = (6)^3 = 216 \text{ cm}^3$

Example 32: Find the volume of a cuboid whose length = 6 cm, breadth = 4 cm, height = 3 cm.

Solution: l = 6 cm, b = 4 cm, h = 3 cm

Volume = $l \times b \times h = 6 \text{ cm} \times 4 \text{ cm} \times 3 \text{ cm} = 72 \text{ cm}^3$

Example 33: What will happen to the volume of a cube if its edge is increased three times?

Solution: Let the edge of the cube be x cm.

Volume of cube $(V_1) = x^3$ cm³

When the edge of cube is increased three times,

New edge = 3x cm

:. New volume $(V_2) = (3x)^3 = 27x^3 \text{ cm}^3$

 \Rightarrow Volume $V_2 = 27 \times (Volume \ V_1)$

Hence, the volume becomes 27 times the volume of the original cube.

Example 34: Water is pouring into a cuboidal reservoir at the rate of 60 litres per minute. If the volume of reservoir is 108 m³, find the number of hours it will take to fill the reservoir.

Solution: Volume of water to be stored in reservoir = 108 m^3

 $= (108 \times 1000) l$

 $[:: 1 \text{ m}^3 = 1000 l]$

= 1.08,000 litres

Since, 60 litres of water is collected in reservoir in 1 minute.

So, 1 litre of water will be collected in the reservoir in $\frac{1}{60}$ min.

 \therefore 1,08,000 litres of water will be collected in the reservoir in $\frac{1}{60} \times 1,08,000$ min, *i.e.*, 1,800 min.

$$1,800 \text{ min} = \frac{1,800}{60} \text{ hours} = 30 \text{ hours}$$

So, it will take 30 hours to fill the reservoir.

Example 35: An open rectangular tank when measured from outside is 2.25 m long, 2.12 m wide and 110 cm deep. It is made up of iron which is 3.5 cm thick. Find the capacity of the tank and the volume of iron used.

Solution: External length of tank =
$$2.25 \text{ m} = (2.25 \times 100) \text{ cm}$$

$$= 225 \text{ cm}$$

External width of tank = $2.12 \text{ m} = (2.12 \times 100) \text{ cm} = 212 \text{ cm}$

External depth of tank = 110 cm

$$\therefore$$
 External volume of tank = 225 cm \times 212 cm \times 110 cm

$$=52,47,000$$
 cm³

Internal length of tank =
$$\{225 - (3.5 \times 2)\}$$
 cm

$$=(225-7)=218$$
 cm

Internal width of tank =
$$\{212 - (3.5 \times 2)\}$$
 cm

$$=212-7=205$$
 cm

$$= (110 - 3.5) \text{ cm} = 106.5 \text{ cm}$$

Internal volume of tank = 218 cm
$$\times$$
 205 cm \times 106.5 cm

$$=47,59,485$$
 cm³

$$= (52,47,000 - 47,59,485) \text{ cm}^3$$

$$=4,87,515$$
 cm³

Example 36: If the total surface area of a cube is
$$32\frac{2}{3}$$
 m², find the volume of the cube.

Solution: If x metres is side of the cube, then surface area =
$$(6x^2)$$
 m² = $32\frac{2}{3}$ m²

$$\Rightarrow 6x^2 = \frac{98}{3}$$

$$\Rightarrow 6x^2 = \frac{98}{3}$$
 $\Rightarrow x^2 = \frac{98^{49}}{3 \times 3} = \frac{49}{3 \times 3}$

$$\Rightarrow x = \frac{7}{3} \text{ m}$$

Thus, the volume of the cube =
$$x^3 = \left(\frac{7}{3}\right)^3 = 12\frac{19}{27}$$
 m³.

Example 37: The dimensions of a cuboid are in the ratio 3:2:1 and the total surface area is 2,816 cm². Find its volume.

Solution: We have the dimensions in the ratio
$$3:2:1$$
.

Let the length, breadth and height be
$$3x$$
, $2x$ and x respectively.

Then, the total surface area of the cuboid =
$$2(lb + bh + hl)$$

$$= 2 \{(3x \times 2x) + (2x \times x) + (x \times 3x)\}\$$

$$= 2 \{ (6x^2) + (2x^2) + (3x^2) \}$$

$$=2\times11x^{2}=22x^{2}$$

Now, $22x^2 = 2,816$ [: Total surface area given is $2,816 \text{ cm}^2$.]

$$\Rightarrow$$
 $x^2 = \frac{2,816}{22} = 128 \text{ cm}^2$

$$\Rightarrow$$
 $x = \sqrt{128} \text{ cm} = 11.31 \text{ cm}$

Now, the dimensions of the cuboid are given by:

length =
$$3x = 3 \times 11.31 \text{ cm} = 33.93 \text{ cm}$$

breadth =
$$2x = 2 \times 11.31 \text{ cm} = 22.62 \text{ cm}$$

$$height = x = 11.31 cm$$

$$\therefore$$
 Volume = $l \times b \times h = 33.93 \times 22.62 \times 11.31 = 8,682.94 \text{ cm}^3$

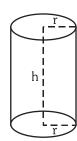
Volume Of A Cylinder

Let three be a cylinder with radius r unit and height h unit.

Volume of the right circular cylinder
$$=$$
 Area of the base \times height

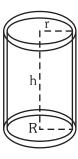
=
$$(\pi r^2 \times h)$$
 cubic units

$$=\pi r^2 h$$
 cubic units



Volume of a hollow cylinder: Let there be a hollow cylinder with external and internal radii R and r respectively and height h.

Volume of the hollow cylinder = External volume – Internal volume =
$$(\pi R^2 h - \pi r^2 h)$$
 cubic units = $\pi h (R^2 - r^2)$ cubic units



Example 38: Find the volume of a cylinder, circumference of whose base is 110 cm and height is 20 cm.

Solution: Let radius of the base of the height circular cylinder be r cm. Then

Circumference =
$$110 \text{ cm}$$

$$\Rightarrow r = \frac{110}{2\pi} = \frac{110 \times 7}{2 \times 22} = \frac{35}{2} \text{ cm}$$

So, the radius of the cylinder is
$$\frac{35}{2}$$
 cm.

Thus, volume of the cylinder =
$$\frac{22}{7} \times \frac{35}{2} \times \frac{35}{2} \times 20 \text{ cm}^3$$

= 19,250 cm³.

Example 39: The volume of one metre rod is 1,386 cm³. Find its diameter.

Solution: Length of rod
$$= 1 \text{ m} = 100 \text{ cm}$$

Volume of rod
$$= 1,386 \text{ cm}^3$$

or,
$$\pi r^2 h = 1{,}386$$

$$\Rightarrow \frac{22}{7} \times r^2 \times 100 = 1{,}386$$

$$\Rightarrow r^2 = \frac{1386 \times 7}{22 \times 100}$$

$$\Rightarrow r^{2} = \frac{1386 \times 7}{22 \times 100}$$

$$\Rightarrow r^{2} = \frac{441}{100} = \left(\frac{21}{10}\right)^{2}$$

$$\Rightarrow$$
 $r = \frac{21}{10} = 2.1 \text{ cm}$

Diameter of the rod = $2 \times r = 2 \times 2.1 = 4.2$ cm.

- **Example 40**: The volume of a metallic cylindrical pipe is 748 cm³. Its length is 14 cm and its external radius is 9 cm. Find its thickness.
- Solution: Let the inner radius of the cylindrical pipe be r and the outer radius be R. Outer radius, R = 9 cm

Height,
$$h = 14$$
 cm

Volume of the pipe =
$$748 \text{ cm}^3$$

Or,
$$748 = \pi R^2 h - \pi r^2 h$$

$$\Rightarrow 748 = \pi h (R^2 - r^2)$$

$$\Rightarrow 748 = \frac{22}{7} \times 14 \times (9^2 - r^2)$$

$$\Rightarrow 748 = 44 \times (81 - r^2)$$

$$\Rightarrow \frac{748}{44} = 81 - r^2$$

$$\Rightarrow$$
 17 = 81 - r^2

$$\Rightarrow r^2 = 81 - 17$$

$$\Rightarrow$$
 $r^2 = 64$

$$\Rightarrow$$
 $r = 8 \text{ cm}$

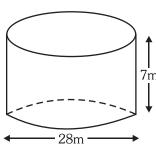
- Thickness = Outer radius Inner radius = 9 cm - 8 cm = 1 cm
- Thickness of the cylindrical pipe is 1 cm.
- Example 41: How many cubic metres of earth must be dug out to sink a well of 7 metres deep and diameter 28 m? Find the cost of plastering its curved surface at ₹ 2.50 per sq m.
- Volume of earth dug out to sink a well = $\pi r^2 h$ Solution:

$$= \left\{ \frac{22}{7} \times \left(\frac{28}{2} \right)^{14} \right)^{2} \times 7 \right\} \text{ m}^{3} \quad [\because r = \frac{28}{2} \text{ m}]$$

$$= \left(\frac{22}{\cancel{7}} \times 14 \times 14 \times \cancel{7}\right) \text{m}^3$$

$$= (22 \times 14 \times 14) \text{ m}^3 = 4,312 \text{ m}^3$$

Now, the area of the curved surface of the well



$$=2\pi rh = \left(2 \times \frac{22}{2} \times 14 \times 7\right) m^2 = (2 \times 22 \times 14) m^2$$

 $=616 \text{ m}^2$

... Cost of plastering the curved surface at the rate of $\stackrel{?}{=} 2.50$ per sq m. $= \stackrel{?}{=} 2.50 \times 616 = \stackrel{?}{=} 1.540$.

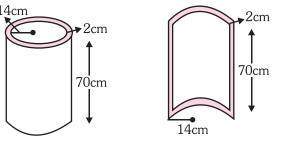


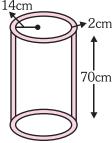
1. Find the volume of a cuboid whose dimensions are:

- (a) length = 11 cm, width = 13 cm and length = 9 cm
- (b) length = 8 cm, width = 4.3 cm and height = 20 cm
- 2. Find the volume of a cube whose side is:
 - (a) 5.4 cm

(b) 7.1 cm

- (c) 4 m
- **3.** Find the volume of the cylinder whose radius is 0.7 cm and height is 1.4 cm.
- **4.** A brick has dimension $10 \text{ cm} \times 6 \text{ cm} \times 5 \text{ cm}$. If 1 cm^3 of brick material weighs 5 g, find the weight of 8 bricks.
- **5.** What happens to the volume of a cube, if the length of its edge becomes—(a) double (b) half.
- **6.** How many soap cakes each of dimensions $7 \text{ cm} \times 5 \text{ cm} \times 2.5 \text{ cm}$ can be placed in a box of dimensions $56 \text{ cm} \times 40 \text{ cm} \times 25 \text{ cm}$?
- 7. The dimensions of a cuboid are in the ratio 4:2:1 and the total surface area is $2,800 \text{ cm}^2$. Find its volume.
- **8.** A cuboidal block of silver is 9 cm long, 4 cm wide, and 3.5 cm high. If beads of volume 1.5 cm³ each are made from it, find the number of beads that can be made from the block.
- **9.** The thickness of a hollow metal cylinder is 2 cm. It is 70 cm long with outer radius of 14 cm. Find the volume of the metal required to make the cylinder assuming that it is open at both ends. Also, find its weight if the metal weighs 8 g per cu cm.





- **10.** The radius and height of a cylinder are in the ratio 5 : 7, and its volume is 550 cm³. Find its radius and height.
- 11. The curved surface area of a cylinder is 4,800 cm² and the circumference of its base is 120 cm. Find the volume of the cylinder.
- 12. The ratio between the curved surface area and the total surface area of a right circular cylinder is 1:2. Find the volume of the cylinder, if its total surface area is 616 cm^2 .
- **13.** How many cubic metres of earth must be dug out to sink a well of 22.5 m deep and diameter 7 m? Also, find the cost of plastering the inner curved surface at ₹ 3 per sq metre.
- 14. A hollow cylindrical tube, open at both ends is made of 2 cm thick iron. If the external diameter is 50 cm and the length of the tube is 140 cm, find the volume of iron in it. Find the weight of the tube, if 1 cm^3 of metal weighs 8 g.

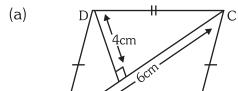
SUMMARY OF THE CHAPTER

- Diagonals of a rhombus bisect each other at right angles.
- A trapezium is a quadrilateral in which one pair of opposite sides are parallel.
- The distance between the two parallel sides of the trapezium is the 'altitude' or 'height' of the trapezium.
- The surface area of every solid is the sum of the areas of all the faces or surfaces that enclose the solid.
- The magnitude of the space occupied by a solid figure is called its volume.



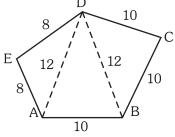
Review Of The Chapter (Task For Summative Assessment)

Find the area of the following figures: 1.



(b) U 12cm -14cm

- 2. In a quadrilateral ABCD as shown in the figure, $AB \parallel CD$ and $AD \perp AB$. If AB = 18 m, DC = BC = 13 m, find the area of the quadrilateral. 18m
- The dimensions of a cuboid are in the ratio 5:3:1 and its total surface area is 414 m². Find the 3. dimensions of the cuboid.
- 4. Find the area of the polygon ABCDE whose measurements in metre are given in the adjoining figure.



13m

- The volume of a cuboid is 36 cm³ and its base area is 18 sq cm. What is its height? **5.**
- Find the volume of the wood used to make a closed box of outer dimensions $60 \text{ cm} \times 45 \text{ cm} \times 32$ 6. cm, the thickness of wood being 2.5 cm all around.
- 7. A cylindrical vessel open at the top has a diameter of 20 cm and a height of 14 cm. Find the cost of tin plasting it on the inside at the rate of \mathbb{Z} 1 per 100 cm².
- The area of the circular base of a cylinder is 616 m² and its height is 3 m. Find the curved surface 8. area of the cylinder.
- 9. One cubic metre of iron is melted and drawn into wire of diameter 4 cm. Find the length of the wire.
- **10.** A room is 7 m long, 7 m wide and 3.5 m high. Find the cost of covering its walls by a paper of 50 cm wide at the rate of \mathbf{T} 1.50 per metre.



Multiple Choice Questions (MCQs)

(Task For Formative Assessment)

The side (s) of a rhombus with diameters x and y is given by : 1.

(a) $\sqrt{x^2 + y^2}$ (b) $\sqrt{\frac{x^2}{2} + \frac{y^2}{2}}$ (c) $\frac{1}{4}\sqrt{x^2 + y^2}$ (d) $\frac{1}{2}\sqrt{x^2 + y^2}$

2. Area of an equilateral triangle is:

3. Three cubes of side 4 cm each are joined end to end to form a cuboid. The surface area of the resulting cuboid and total surface area of the three cubes are in the ratio:

(a) 9:7

(b) 7:9

(c) 7:3

(d) 1:4

If the volumes of two cubes are in the ratio 8:1, the ratio of their edges is: 4.

(a) 8:1

6.

(b) $2\sqrt{2}:1$

(c) 2:1

(d) 1:8

- The sum of areas of all faces (excluding top and bottom) of a cuboid is the 5.

(b) surface area (c) lateral surface area (d) none of these

The maximum length of a rod that can be kept in a cuboidal box of dimensions $12 \text{ cm} \times 9 \text{ cm} \times 8$ cm is:

(a) 13 cm

(b) 17 cm

(c) 18 cm

(d) 19 cm



Aim: To understand the curved surface area, total surface area and volume of a closed cylinder by an activity method.

Materials required: Chart paper, geometry box, a pair of scissors, and a cellotape or fevicol. **Process:** Make a cylinder of radius 3.5 cm and height 10 cm.

Step I: Take a chart paper and draw two circles of radius 3.5 cm each shown in the figure. Cut the two circles along the dotted lines in the flap.



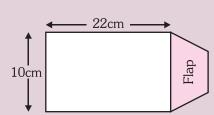


Step II: Draw a rectangle whose length = circumference of the circle (given)

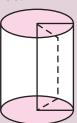
 $= 2 \times \pi \times r$ $=2\times\frac{22}{7}\times3.5$

 $=22\,\mathrm{cm}$

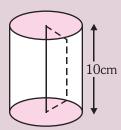
And, breadth = 10 cm as shown in the figure. Cut the rectangle along the flap.



Step III: Roll the rectangle and join the flap using cellotape, as shown in the figure. The cylinder is open at both ends.



Step IV: Fix the two circles at the open ends of the cylinder using cellotape as shown.



Observations:

- 1. The curved surface area of the cylinder = area of the rectangle = length \times breadth = 22×10 = 220 cm^2
- 2. The sum of the area of the bases = $2 \times$ area of the circle of radius 3.5 cm = $2 \times \pi \times r^2$ = $2 \times \frac{22}{7} \times (3.5)^2$ = 77 cm^2
- 3. Total surface area of the cylinder = curved surface area + areas of two bases = $220~\text{cm}^2 + 77~\text{cm}^2$ = $297~\text{cm}^2$
- 4. The volume of the cylinder = area of the base \times height = $\pi r^2 \times h$ = $\frac{22}{7} \times (3.5)^2 \times 10$ = 385 cm^3



Data Handling

Sequence of the Chapter

- Presentation Of Data
- Grouping Of Data
- ✓ Bar Graph (Or Column Graph)
- ✓ Double Bar Graphs
- ✓ Pie Chart

INTRODUCTION

We are living in a world of information and technology, world of print and electronic media. In everyday life, we come across the numerical data in the newspapers, magazines, television, etc. The data may relate to anything—weather, share market, population, cost of living, pollution, growth of an industry or profit of an industrial group, etc. Numerical data helps us to draw inferences. This process of collecting the data and drawing inferencing is called 'statistics'. Thus,

Statistics is the branch of mathematics which deals with collection, classification, analysis and interpretation of data for making useful inferences.

In this chapter, we'll learn about raw and grouped data, making frequency table from raw data, reading and interpreting bar graphs, line graphs, double bar graphs, pie charts. The process of collecting data, presenting the data, making inferences is dealt in **data handling**.

PRESENTATION OF DATA

We all know that data is the given or available information. Raw data is like an unsolved puzzle which makes no meaningful impression at the first glance other than being a collection of information. So, raw data needs to be arranged in a meaningful format so that the salient features of the given information can be evaluated and studied. It can be arranged in an ascending or descending order, which is then called an array or arranged data.

Data is of two types:

1. Discrete Data

The data which can take only whole number values is called **discrete data**. For example, number of students in different classes: 10, 15, 22, 30, etc.

2. Continuous Data

The data which can take any value between two whole numbers is called **continuous data**. *For example*, height of students: 146 cm, 152.50 cm, 148.37 cm, 156.25 cm. Before we learn to tabulate data, let's learn the following terms:

- (i) Range: The difference between the greatest and small observations is called the range of the data.
- (ii) Frequency: Sometimes, observations can be too many and it becomes a cumbersome and tedious task to arrange the terms in ascending or descending order. To arrange such data in a more meaningful way, count the number of times a particular observation occurs and tabulate it. Frequency is defined as the number of times a particular observation occurs in a data.
- (iii) Tally Marks: Tally marks are used to count the frequency. The symbol | is used to mark each frequency. Tally marks are recorded in groups of 5. The fifth tally mark is drawn by crossing the first four tally marks diagonally like \| \| \| \| \| \| \.

GROUPING OF DATA

Let us observe the marks obtained by 25 students in Mathematics test as follows : 25, 32, 41, 33, 30, 31, 50, 32, 28, 30, 48, 50, 30, 48, 41, 45, 25, 28, 31, 45, 28, 41, 45, 25, 45

We observe that there are few students who get same marks, *e.g.*, 50 is obtained by 2 students, 31 is obtained by 2 students, 25 is obtained by 3 students, etc. Let us represent them in a frequency distribution table as given below:

Marks	Tally Marks	Frequency
25		3
28	Ш	3
30	Ш	3
31	П	2
32	П	2
33		1
41	Ш	3
45		4
48	П	2
50		2
		Total = 25

Here, we see that the lowest mark is 25 and highest mark is 50.



In each of the classes, the upper limit is not included, e.g., 30 is not included in 20-30, but is included in 30-40.

We can further group them into classes as given ahead .



Classes	Tally Marks	Frequency
20 – 30	<u> </u>	6
30 – 40	THL 111	8
40 – 50	 	9
50 – 60		2

We have formed four groups: 20 to 30, 30 to 40, 40 to 50 and 50 to 60.

Each of these groups is called a **class interval** or simply a **class**. The class intervals are written as 20-30, 30-40, etc. This type of classes is called **continuous classes**. In class 20-30, 20 is called **lower class limit** and 30 is called **upper class limit**. The average of upper and lower limits is called **class-mark**, *e.g.*,

Class-mark of 20-30 is
$$\frac{20+30}{2} = \frac{50}{2} = 25$$
.

The difference between upper and lower class limits is called **class-size** or **width** of the class. Here, class-size is 30 - 20 = 10.

Example 1: The shoe-sizes of 40 students of a class are given below:

6 5 9 9 7 8 5 7 5 8 6 7 9 9 8 7 8 7 8 9 5 6 9 5 7 7 6 5 7 5 8 5 5 6 5 6 5 6 8 9

Make a grouped frequency distribution table from this data.

Solution: Highest observation = 9, Lower observation = 5

 \therefore Range of the data = 9 - 5 = 4 Let us divide the data into groups of size 2.

Classes	Tally Marks	Frequency
4 – 6	M M I	11
6 – 8		15
8 – 10	M M IIII	14
		Total = 40

Example 2: Following table is showing the distribution of marks of students:

Marks (Class-internals)	25–35	35–45	45–55	55–65	65–75
No. of students	5	1	5	9	4

Read the data carefully and answer the following questions:

- (a) What is the lower limit of the class 45-55?
- (b) What is the upper limit of the class 65-75?

(c) What is the class-mark of the second class-interval (35-45)?

(d) What is the class-size?

Solution:

(a) The lower limit of the class 45-55 is 45.

(b) The upper limit of the class 65-75 is 75.

(c) The class-mark of the second class interval (35-45)

$$=\frac{35+45}{2}=\frac{80}{2}=40.$$

(d) Class-size = upper class limit - lower class limit

$$=75-65=55-45=10.$$



1. A survey was conducted in 25 families of a locality. Following are the number of children in these families. Prepare a frequency distribution table of the data.

2, 2, 3, 1, 1, 1, 2, 3, 2, 2, 1, 1, 2, 2, 1, 1, 3, 2, 2, 1, 1, 2, 2, 2, 2

2 Complete the following frequency distribution table:

(a)	Marks	Tally Marks	Frequency
	3		
	5	M III	
	7	HH HH I	
	9	1111	
	11	144	
	13	 	
	15	M M II	

(b)	Heights (in cm)	Tally Marks	Frequency
	40–50		
	50–60	M4 M4 II	
	60–70	M4	
	70–80	 	
	80–90	111	
	90–100		

3. Following data represents the marks of a group of 40 students in examination. Represent the data by a frequency distribution table using the same class-size, one such class being 20-30 (30 not included):

18, 27, 36, 18, 43, 31, 37, 41, 28, 35, 39, 29, 19, 24, 18, 27, 46, 36, 36, 21, 13, 26, 23, 32, 18, 30, 19, 31, 17, 34, 22, 19, 27, 42, 40, 37, 37, 25, 23, 27

4. Study the following frequency distribution table of daily income of 550 workers of a factory and answer the following questions:

Class-interval (Daily income in ₹)	Frequency (Number of workers)
100 – 125	45
125 – 150	25
150 – 175	55
175 – 200	125
200 – 225	140
225 – 250	55
250 – 275	35
275 – 300	50
300 – 325	20
	Total = 550

- (a) Which class has the highest frequency?
- (b) Which class has the lowest frequency?
- (c) Which two classes have the same frequency?
- 5. Construct a frequency distribution table for the following weights (in g) of 35 oranges using the equal class-intervals, one of them is 50-55 (55 is not included).
 - 30, 40, 45, 32, 43, 50, 55, 70, 61, 62, 70, 53, 62, 37, 52, 42, 50, 35, 53, 55, 73, 74, 70, 65, 46, 45, 68, 60, 59, 58, 34, 74, 62, 35, 70
 - (a) What is the class-mark of the class-interval 40-45?
 - (b) What is the range of the above weights?
 - (c) How many classes are there?
- 6. Construct a frequency distribution table of class-size 10 for the following data:
 - 35, 11, 22, 28, 7, 10, 15, 5, 21, 42, 29, 31, 16, 25, 24, 18, 10, 11, 12, 14, 3, 8, 6, 11, 23, 25, 45, 33, 16, 49, 22, 35, 9, 14, 15, 11, 19, 30, 27, 8

BAR GRAPH (OR COLUMN GRAPH)

Bar graph is a simple pictorial representation of data consisting of numbers. In the bar graph, the bars are rectangles of uniform width drawn with equal spacing between them. The bars may be drawn vertically or horizontally. The bar graphs are usually used to represent ungrouped data.

Drawing A Bar Graph

To draw a bar graph, follow the given steps:

- Step I: Take a graph paper and draw a horizontal line *OX* and a vertical line *OY*. These lines are called the X-axis and Y-axis respectively.
- Step II: Mark points at equal intervals along the X-axis. Below these points write the names of the data items whose values are to be plotted.
- Step III: Choose a suitable scale on Y-axis. On that scale find the heights of the bars for the given numerical values.
- Step IV: Mark off these heights parallel to the Y-axis from the points taken in step II.

Step V: On the X-axis, draw bars of equal width for the heights marked in step IV. The bars should be centered on the points marked on the X-axis. These bars represent the given numerical data.

Reading A Bar Graph

We can draw certain conclusion after studying a bar graph. The process of obtaining various information related to the given data from a bar graph is known as **interpretation of the bar graph**.

Example 3: The number of suits sold in a ready-made garments shop from Monday to Saturday of a week is given in the following table:

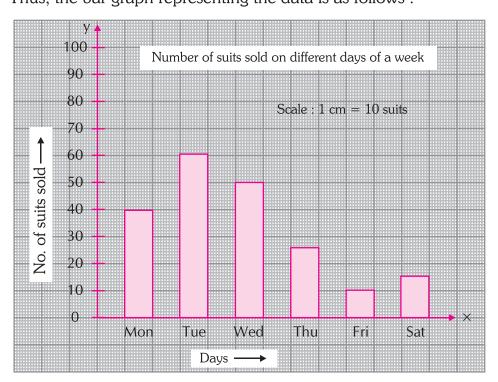
Days	Mon	Tue	Wed	Thu	Fri	Sat
No. of suits sold	40	60	50	25	10	15

Represent this data by a bar graph.

Solution: Step I: Draw two mutually perpendicular rays. Name the horizontal one as *OX* and vertical one as *OY*.

Step II: On OX, label 'Days' and on OY, label 'No. of suits sold'.

Step III: Let us choose the scale on OY as 1 cm = 10 suits. Thus, the bar graph representing the data is as follows:



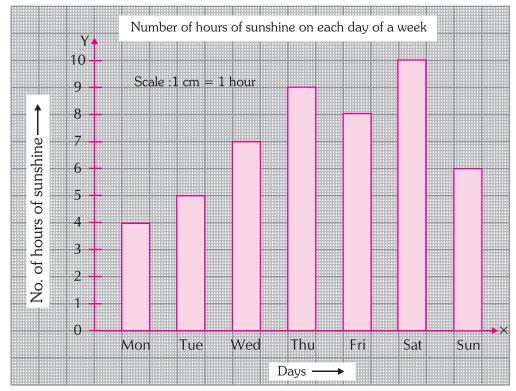
Example 4: The following table shows the number of hours of sunshine on each day of week.

Days	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Hours of sunshine	4	5	7	9	8	10	6

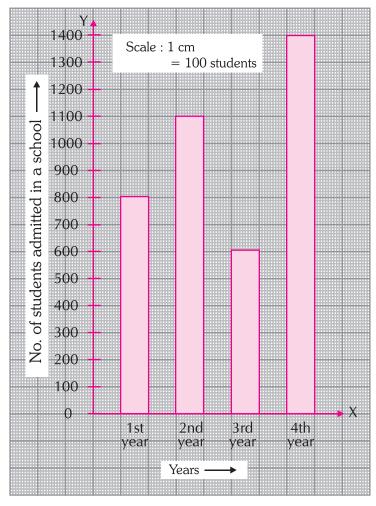
Draw a bar graph for the above data.

Solution:

Let us choose a suitable scale on Y-axis. 1 cm = 1 hour



Example 5: Read the following bar graph and answer the following questions:



- (a) What information does the graph represent?
- (b) In which year the number of admissions was the maximum?
- (c) How many students were admitted in the second year?
- (d) How many students were admitted by 3rd year in total?
- (e) What is the increase in number of admission in 4th year with respect to 3rd year?

Solution:

- (a) The graph represents the number of students admitted in a school in four different years.
- (b) In 4th year, the number of admissions was maximum.
- (c) 1100 students were admitted in the second year.
- (d) Total number of students admitted by 3rd year = total number of students admitted in 1st year, 2nd year and 3rd year = 800 + 1100 + 600 = 2500.
- (e) Increase in the number of admissions in 4th year with respect to 3rd year = number of students in 4th year number of students in 3rd year = 1400 600 = 800.



1. The following table shows the favourite sports of 250 students of a school:

Sports	Football	Cricket	Table-tennis	Bad-minton
No. of students	80	55	50	65

Construct a bar graph to represent the given data.

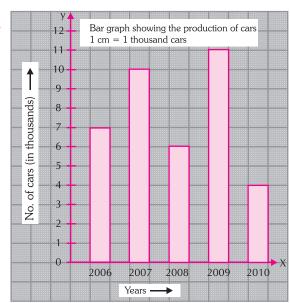
2. The number of students in a hostel from different states is given below:

States	Bihar	Punjab	Orissa	Maharashtra	Madhya Pradesh
No. of students	10	15	30	40	25

Construct a bar graph to represent the given data.

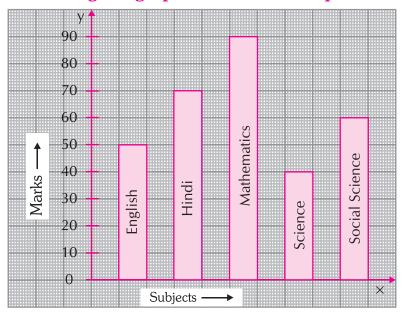
3. Read the bar graph given alongside and answer the following questions:

- (a) What is the general information given by the bar graph?
- (b) In which year the production of cars was the maximum? What is the maximum number of cars produced during that year?
- (c) In which year the production of cars was minimum? What is the minimum number of cars produced during that year?
- (d) What is the difference between the maximum and minimum production of cars?
- (e) What is the ratio of the production of cars during the year 2006 to that in the year 2010?





4. Study the following bar graph and answer the questions given below:



- (a) What information is depicted in the graph?
- (b) In which subject has the student scored the maximum marks?
- (c) In which subject has the student scored least marks?
- (d) What is the score of the student in Social Science?
- (e) What is his total score out of 500?

5. Draw a bar graph representing the following data:

Monthly wages (in rupees)	Number of teachers
1000 – 1100	5
1100 – 1200	15
1200 – 1300	20
1300 – 1400	10
1400 – 1500	9
1500 – 1600	8
1600 – 1700	25
1700 – 1800	20
1800 – 1900	10
1900 – 2000	15



A bar graph can also represent two sets of data simultaneously. Such a bar graph is called a **double bar graph**. It is very useful in comparing two sets of data.

- **Example 6:** The table given ahead gives the marks of a students in four tests in two subjects-Mathematics and English.
 - (a) Represent the given data by a double bar graph.
 - (b) From the graph, find in which subject the student performs better.

Tests	Test I	Test II	Test III	Test IV
Marks in Mathematics	20	25	30	15
Marks in English	25	28	20	15

Solution:

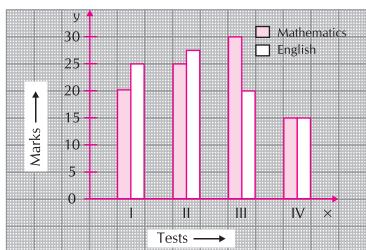
(a) The graph is made using the given steps:

Step I: The X-axis and Y-axis are drawn. On the X-axis, 4 points are marked at equal distances. These four points represent the four tests.

Step II: On the Y-axis, 30 points are marked at equal distances to represent marks. In the given graph, the points are marked at a distance of 5 units.

Step III: On the X-axis, on either side of the point marked for Test I, two bars are drawn next to each other with no gap (space) in between. The first bar represents the marks in Mathematics in Test I and the second bar represents the marks in English in Test I. The height of the bars corresponds to the marks in the respective subjects. The two bars are shaded differently to differentiate between the marks of the two different subjects. The different shades for the two subjects are mentioned on the graph.

Step IV: The bars for the marks in the two subjects for Tests II, III and IV are drawn in the same way.



(b) In Test IV, the marks in both the subjects are the same. In Test III, the bar representing English is shorter than the bar representing Mathematics. But the remaining two bars representing English are longer than the bars representing Mathematics. Thus, we can say that the student has performed better in English as compared to Mathematics.



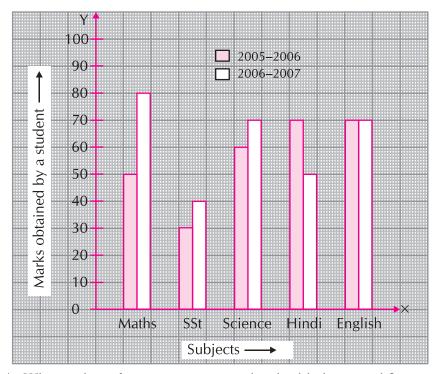
1. The following data shows the percentage wins in ODI by top 8 cricket teams. Represent it in a double bar graph.

Teams	South Africa	Australia	Sri Lanka	New Zealand	England	Pakistan	West Indies	India
From Champions Trophy to World Cup 06	75%	61%	54%	47%	46%	45%	44%	43%
Last 10 ODI in 2007	78%	40%	38%	50%	40%	44%	30%	56%

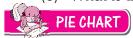
2. The following data shows the temperatures in °C and the humidity percentages on five days in a week. Represent it in a double bar graph.

Days	1	2	3	4	5
Temperature in °C	30	28	32	29	33
Humidity %	95	94	92	88	90

3. Study the following bar graph and answer the questions given below:



- (a) What is the information given in the double bar graph?
- (b) In which subject has the performance improved the most?
- (c) In which subject has the performance deteriorated?
- (d) In which subject is the performance at par?
- (e) What is the difference in the average scores for the year 2005-2006 and 2006-2007?



A pie chart is a pictorial representation of numerical data in a circle divided into sectors. Each sector is proportional to the data represented by it. A pie chart is also called the **circle graph**.

Drawing A Pie Chart

To draw a pie chart, follow the given steps:

Step I: Find ratio of each component to the total fraction.

Find what fraction or percentage in each component of the total.

Find the central angle of each component, where central angle of a component

$$= \left(\frac{\text{Value of the component}}{\text{Total value}} \times 360\right)^{\circ}$$

Step III: Draw a circle of any radius.

Step IV: Draw a horizontal line as the radius of the circle.

Step V: Starting with this horizontal radius, draw radii making central angles corresponding to the values of the respective components calculated in step II. Repeat this process for all the components of the given data. These radii will divide the circle into number of sectors.

Step VI: To beautify use different colours or shadings to denote various components.

Reading A Pie Chart

The process of obtaining various information related to the given data from a given pie chart is known as **interpretation of a pie chart**.

To read a pie chart, we need to know the following results:

(i) Value of a component = $\frac{\text{Central angle of the component}}{\text{Central angle of the component}} \times \text{Total of the component values}$

(ii) Percentage value of a component = $\frac{\text{Central angle of the component} \times 100}{\text{Central angle of the component}}$

Example 7: A person spends his salary in the following manner:

Total salary = ₹12000 Food ₹ 1800 = ₹1200 Clothing House rent = ₹ 3000

Education of children = ₹1200 Miscellaneous expenditure = ₹2400

= ₹2400 Savings

Make a pie chart exhibiting the above information.

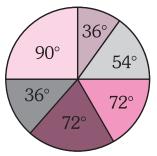
Solution: To find the degree measure of sectors representing different values, we first prepare the following table:

Items	Expenditure (in ₹)	Percentage	Degree $100\% = 360^{\circ}$ $\Rightarrow 1\% = 3.6^{\circ}$
Food	1800	$\frac{1800}{12000} \times 100 = 15\%$	$15 \times 3.6 = 54^{\circ}$
Clothing	1200	$\frac{1200}{12000} \times 100 = 10\%$	$10 \times 3.6 = 36^{\circ}$

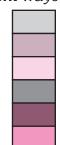
House rent	3000	$\frac{3000}{12000} \times 100 = 25\%$	$25 \times 3.6 = 90^{\circ}$
Education	1200	$\frac{1200}{12000} \times 100 = 10\%$	$10 \times 3.6 = 36^{\circ}$
Miscellaneous Expenditure	2400	$\frac{2400}{12000} \times 100 = 20\%$	$20 \times 3.6 = 72^{\circ}$
Savings	2400	$\frac{2400}{12000} \times 100 = 20\%$	$20 \times 3.6 = 72^{\circ}$

Now, draw a circle of any radius and divide it into six sectors with the sector angles measuring 54°, 36°, 90°, 36°, 72° and 72°.

Shade the six sectors in different ways as shown below.



Scooter



- 1. Food
- 2. Clothing
- 3. House rent
- 4. Education
- 5. Miscalleneous Expenditure

Cycle 90°

120°

Train

Bus

60°

Scooter

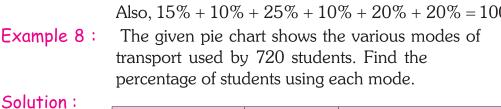
50°

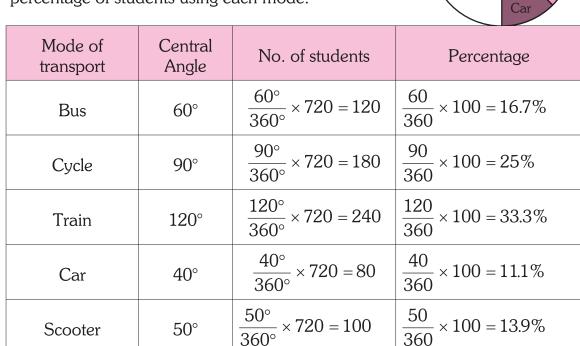
40°

6. Savings

Check: $54^{\circ} + 36^{\circ} + 90^{\circ} + 36^{\circ} + 72^{\circ} + 72^{\circ} = 360^{\circ}$ Also, 15% + 10% + 25% + 10% + 20% + 20% = 100%

Example 8:





50°

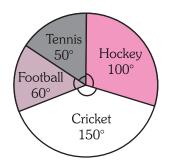


1. The following data shows the number of students offered different subjects in a school:

Subjects	No. of students
English	45
Mathematics	60
Physics	20
Chemistry	30
Economics	25
	Total = 180

Represent the above data by a pie chart.

- 2. The pie chart given alongside represents the amount spent by a sports club in a year on four different sports.
 - (a) If the club spends a total of $\ref{1,08,000}$ in a year, find the amount spent on each sport.
 - (b) If the club spends ₹ 20,000 on hockey in a year, find the total amount spent by the club on sports in a year.



3. The following data relates to the cost of construction of a house in New Delhi. Represent the data in a pie chart.

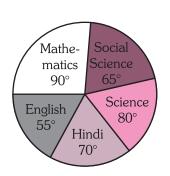
Items	Cement	Steel	Bricks	Timber	Labour	Misc.
Expenditure	30%	10%	10%	15%	25%	10%

4. The number of hours spent by a class VIIIth student on various activities are given below:

Activities	School	Sleep	Homework	Play	Others
No. of hours	8	7	4	3	2

Represent the above information with the help of a pie chart.

- 5. The adjoining pie chart gives the marks scored in an examination by a student in Hindi, English, Mathematics, Social Science and Science. If the total marks obtained by the students were 540, answer the following questions:
 - (a) In which subject student scored 105 marks.
 - (b) How many more marks scored in Mathematics than in Hindi?
 - (c) Did student score more marks in Social Science and Mathematics than in Science and Hindi?



SUMMARY OF THE CHAPTER

- Statistics is the branch of mathematics which deals with collection, classification, analysis and interpretation of data for making useful inferences.
- The data which can take only whole number values is called discrete data.
- The data which can take any value between two whole numbers is called continuous data.
- Frequency is defined as the number of times a particular observation occurs in a data. 回
- In a bar graph, the bars are rectangles of uniform width drawn with equal spacing between them. 回
- A pie chart is a pictorial representation of data in a circle divided into sectors.



Review Of The Chapter (Task For Summative Assessment)

The following is the distribution of weights (in kg) of 40 students: 1.

Weights (in kg)	Persons
20 – 30	8
30 – 40	10
40 – 50	12
50 – 60	3
60 – 70	7

Study the table given above and answer the following questions:

- (a) What is the lower limit of the class-interval 30-40?
- (b) Find the class-marks of the classes 40-50 and 60-70.
- (c) What is the class-size?

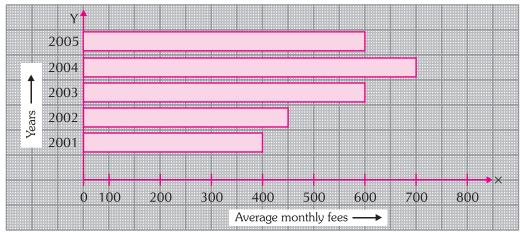
2. Pulse rate (per minute) of 30 persons were recorded as:

61, 76, 72, 73, 71, 66, 78, 73, 68, 81, 78, 63, 72, 75, 80, 68, 75, 62, 71, 81, 73, 60, 79, 72, 73, 74, 71, 64, 76, 71.

Construct a frequency distribution table using class-intervals of equal width, one class-interval being 60-65.

3. The following bar graph represents the average monthly fees charged by the public schools of a city over period of five years-2001 to 2005.

Read the graph and answer the questions given ahead:



- (a) What is the period covered in the bar graph?
- (b) How much was the increase in the fees from 2001 to 2004?
- (c) When was the fees minimum?
- (d) What was the maximum fees?
- (e) Did the fees increase uniformally?

4. The following table shows the subjects preferred by 300 students of a school during a survey. Represent it in a pie chart.

Subjects	Maths	Science	English	Moral Science	Social Science	Total
No. of students	75	90	60	40	35	300

5. The maximum temperatures of five days of a week (in °C) are given below:

Days	Maximum temperatures (°C)
Tuesday	38
Wednesday	42
Thursday	35
Friday	36
Saturday	40

Represent this information using a bar graph.



Multiple Choice Questions (MCQs)

(Task for Formative Assessment)

1.	The number of times a particular entry occurs in an observation is known as :	
	(a) range (b) class-size (c) frequency (d) class-mark	
2.	The mid-value of a class-interval is called :	
	(a) class-limit (b) class-mark (c) class-size (d) range	
3.	The difference between the greatest and the smallest value of an observation is called its:	
	(a) frequency (b) range (c) class-interval (d) none	
4.	The difference between upper and lower class-limits of a group is called:	
	(a) class-interval (b) class-limit (c) class-size (d) class-mark	
5.	The method of conveying numerical information by bars is called:	
	(a) pictograph (b) bar graph (c) piechart (d) stick graph	
6.	The adjoining pie chart shows the interests of the students of a school in different subjects. How many students like languages, if there are 1800 students in the school? (a) 450 (b) 300	
	(a) 450 (b) 390	
	(c) 480 (d) 540	



Aim: To collect data and represent it in a bar graph.

Materials required: White and coloured sheets of paper, scale, graph paper, pencil, a pair of scissors, glue stick.

Process:

Step I: Ask the blood group of (say 40) students of your class. Note the data in a table as shown below:

S.No.	Name of the student	Blood group
1		A, B, AB, O
2		
3		
4		
	•••••	

Step II: Now group these observations in the table given below:

Blood group	Tally marks	Number of students
A		
В		
AB		
0		
Total		

Step III: Draw a bar graph for the table in step II on the white sheet.

Step IV: Cut strips of different colours from the coloured sheets and paste them on the bars.

Observations:

- 1. The most common blood group in your class is
- 2. The least common blood group in your class is _____